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Conception of effective number of lanes as basis of traffic optimization

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Abstract

This article describes the methods of mathematical modeling and simulation of processes of local interaction in transport systems. Described transport flow models, developed by the author, based on extension of the microscopic Treiber's model and on the queuing theory. These models cover more details of behavior of flow participants on signal-controlled crossroads. modification. The article describes a model of traffic on the ring road.

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Traffic modeling, traffic, simulation, queuing theory, vehicle, signal-controlled road intersections, traffic management, ring road, matrix of correspondence, traffic optimization

1. Microscopic model of traffic

The appearance of computers has allowed to produce complex numerical experiments using the imitating modeling of process and has become possible to consider random nature of traffic flow. Modelling is commonly used in cases where studied systems can not be analyzed by direct or formal analytical methods. In microscopic models each vehicle is considered as a separate element of the transport system. It is assumed that the acceleration of the vehicle depends on the neighboring vehicles. The greatest influence on the behavior of the driver has the vehicle situated ahead, «the leader».

The authors developed a microscopic model of traffic flow, extends existing Treiber's «Intellect Driver Model» [1] in case of multilane roads and signal-controlled road intersections. In this model each car have a desired speed in the range $0 \ldots V_{max}$, value d — is the distance between the current vehicle and the next vehicle in front of it, V_n — speed of the current vehicle. Update rules are set by Treiber's model. Treiber's model was extended to the two-dimensional case by introducing the probability characteristics

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of possibility of lane change and necessity of this action, which allowed to consider the behavior of vehicles when turning at a crossroads.

For the considered model was developed complex computer programs BTS SIM, which perform a series of numerical experiments. The microscopic model was used to solve the problem of movement of vehicles on the signal-controlled road intersections. Consider the motion of vehicles at crossroad with predetermined heterogeneous multi-purpose flows. we calculated the optimal duration of the phases of traffic light mode on the grid of values for maximum throughput at crossroad with given intensity of flows at each direction. The resulting phase distribution were tested for resistance to small changes in the input flow rates from the various directions that showed the possibility of working with inexact data.

Let us introduce the concept of «effective number of lanes» which can be illustrated by the following example. Suppose that the vehicle flow, moving along six-lane road arrived to a three-way road intersection (T junction). Assume that the goal of the third of drivers is turning to the left, and the aim of of other is going straight.

In this case, the two leftmost lanes will take vehicles whose goal is to turn left. Thus, at the moment the green light is on for forward movement, the flow of vehicles won't be maximum for the given number of lanes as we might expect, but less by one third, meaning not six thousand but four thousand vehicles per hour.

Let us state the following lemma:

<u>Lemma on the equilibrium maximum capacity on the signal-controlled road intersection</u>. Consider controlled multilane intersection with Poisson traffic. Consider the flow of cars from one direction at a fixed traffic light phase. Let N — number of incoming lanes, N_i — number of lanes at target road i, S — maximum capacity per lane , $\Omega_i(t)$ — vehicle queue from incoming road to the direction i at time t , $\Omega(t)$ — vehicle queue at incoming road at time t, $\Omega_a(t)$ — vehicle queue from incoming road, which can continue their motion to the direction i on current phase at time t. Then expectation of equilibrium maximum capacity on the signal-controlled road intersection on this phase in time T is:

$$E[S_{max} \cdot T] = \int_0^T min(S \cdot N \cdot \frac{\Omega_a(t)}{\Omega(t)}, \sum_i S \cdot N_i \cdot \frac{\Omega_i(t)}{\Omega(t)})dt.$$

Because of this lemma it was obtained the following consequence:

<u>Corollary of effective number of lanes in the absence of asymmetry queues</u>. Consider controlled multilane intersection with Poisson traffic. Consider the flow of cars from one direction at a fixed traffic light phase. Suppose there is no queue at initial time on the crossroad.

Let q to be intensity of the incoming traffic, k — total intensity of vehicles from the incoming traffic, which can continue their motion on current phase, S — maximum capacity per lane, N — number of incoming lanes, N_i — number of lanes at target road i, k_i — total intensity of vehicles from the incoming traffic, which can continue their motion to the direction i on current phase.

Then expectation of equilibrium maximum capacity on the signal-controlled road intersection on this phase is:

$$E[S_{max}] = \min\left(S \cdot N \cdot \frac{k}{q}, \sum_i S \cdot N_i \cdot \frac{k_i}{q}\right).$$

To solve the problem of searching for the expected delay occurring at overcoming of signal-controlled road intersection with a fixed duration phases queuing theory has been applied.

Let's introduce the following abbreviations and symbols:

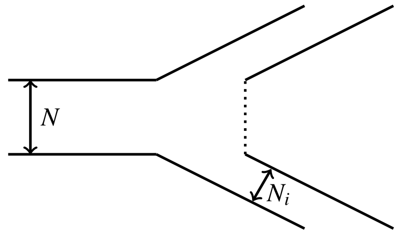

Fig. 1. Maximal equilibrium throughput: illustration

Nomenclature

c	signal cycle
c_i	the duration of the i-st phase
q	traffic arrival flow rate
S	maximum departure flow rate
CPT	Cars per time, the average number of vehicles passing the intersection for 1 second
Q_0	expected overflow queue from previous cycles
Q(t)	vehicle queue at time t

The arrival process is the Poisson process, arriving to traffic light with a fixed time of phases. Total vehicle delay during one signal cycle as a sum of all phase components can be expressed

$$W = \sum_{i=1}^{4} W_i,$$

where W_i - total delay experienced in the $i-$th phase,

$$W_i = \int_{\sum_{j=1}^{i-1} c_j}^{\sum_{j=1}^{i} c_j} \Omega(t)dt.$$

Using the concept of the effective number of lanes, we can put k_i — the corresponding intensity for each flow that can continue movement in each phase. In this case departure flow rate is equal to

$$E[S_i] = S \cdot \frac{k_i}{q}.$$

Total result can be summarized as follows:

$$E[W] = c\Omega_0 + \frac{P_1 c_1^2}{2} + \frac{P_2 c_2^2}{2} + c_2 P_1 c_1 + \frac{P_3 c_3^2}{2} + c_3(P_1 c_1 + P_2 c_2) + \frac{P_4 c_4^2}{2} + c_4(P_1 c_1 + P_2 c_2 + P_3 c_3),$$

where $P_i = q - E[S_i]$ — so-called «surplus flows». Let P_{ij} — surplus flow for the road number i in the phase j, Ω_{i0}— initial queue of vehicles on this road.

Thus we consider expectation of total delay of vehicles for the time of T that is for T/c traffic lights cycles [2]:

$$E[W] = c(\sum_{i=0}^{\frac{T}{c}}(\sum_{j=1}^{4} \max(\Omega_{j0} + \sum(c_i P_{ji})i, 0) + c_1 \sum_{i=1}^{4} P_{i1})(c_1/2 + c_2 + c_3 + c_4)+$$
$$+(c_2 \sum_{i=1}^{4} P_{i2})(c_2/2 + c_3 + c_4) + (c_3 \sum_{i=1}^{4} P_{i3})(c_3/2 + c_4) + (c_4 \sum_{i=1}^{4} P_{i4})(c_4/2)) \cdot \frac{T}{c}.$$

For specific road intersections values P_{ij} can be considered constant. General solve of this task rather cumbersome and partitioned into a lot of specific cases depending of these values.

Let us calculate the total delay of vehicles per traffic light cycle from all directions for the road intersection, calculated at [1].

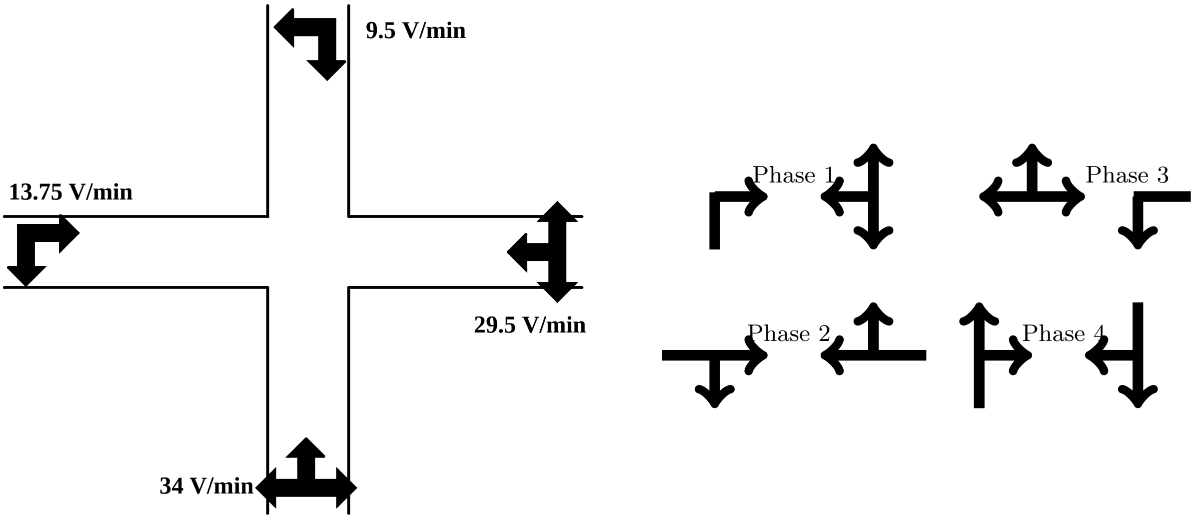

Fig. 2. Crossroad and traffic lights phases

This reduces our problem to the problem of minimizing the total delay per unit of time: $E(W)/T$, i.e., to minimizing the function

$$
\begin{aligned}
F = \;& (c \cdot \tfrac{1}{2}(\tfrac{T}{c} + 1)\tfrac{T}{c}((\max(14c_1 - 26c_2 + 34c_3 - 6c_4, 0) + \\
+ \;& \max(13.75c_1 + 13.75c_2 - 46.25c_3 + 13.75c_4, 0) + \\
+ \;& \max(9.5c_1 + 9.5c_2 + 9.5c_3 - 51.5c_4, 0) + \max(-31.5c_1 + 9.5c_2 - 11.5c_3 + 29.5c_4, 0)) + \\
+ \;& (c_1(5.75)(c_1/2 + c_2 + c_3 + c_4) + c_2(6.75)(c_2/2 + c_3 + c_4) + \\
+ \;& c_3(-14.25)(c_3/2 + c_4) + c_4(-14.25)(c_4/2)) \cdot \tfrac{T}{c})/T
\end{aligned} \tag{1}
$$

on the pyramid $c_1 \geqslant 0$, $c_2 \geqslant 0$, $c_3 \geqslant 0$, $c - c_1 - c_2 - c_3 \geqslant 0$.

In general the solution of this problem depends on the ratio T/c, the number of cycles, but the obtained solution is asymptotic and hardly subject to fluctuations since $T/c = 50$ (1).

c_1^*/c	c_2^*/c	c_3^*/c	c_4^*/c	T/c
0.208502	0.368045	0.229767	0.193685	40
0.209071	0.367474	0.2292	0.194255	50
0.209071	0.367473	0.229201	0.194255	60
0.209071	0.367474	0.2292	0.194255	70
0.209071	0.367473	0.2292	0.194255	80
0.209071	0.367474	0.2292	0.194255	90
0.209071	0.367474	0.2292	0.194255	100

Table 1. Optimal value of c_i^*/c as dependency of cycles count T/c

Comparison of the results of the analytical and microscopic models for the crossroad (Tab. 2, 3) show that the results obtained by the analytical apparatus are correlated with the results of microsimulation and outperform them.

Phase 1, sec	Phase 2, sec	Phase 3, sec	Phase 4, sec	CPT
35	61.5	38.4	32.5	0.761

Table 2. Optimal lengths of phases, obtained analytically

Phase 1, sec	Phase 2, sec	Phase 3, sec	Phase 4, sec	CPT
60	30	45	30	0.737
45	30	30	30	0.737
30	75	45	30	0.737
45	60	45	30	0.736
45	45	45	30	0.735
75	30	45	30	0.735
...	...	...	...	...
45	45	30	90	0.585
30	30	45	90	0.584
30	30	60	90	0.584
45	30	30	90	0.566
30	30	30	90	0.560

Table 3. Dependency of total throughput for crossroad, sorted by CPT in decreasing order, obtained by micromodelling

2. Traffic on the ring road

The following describes the modeling of movement on one-way (uni-directional) ring road. Considered freeway is represented as shown in fig. 3.

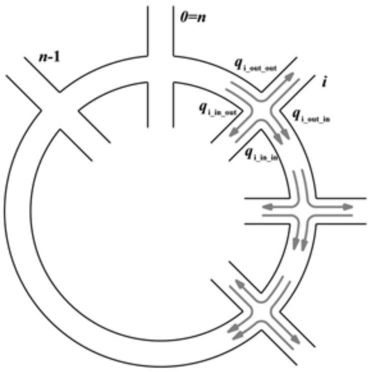

Fig. 3. Scheme of ring road

Here

Nomenclature

q_i	the traffic flow from the node number i to the node number $i+1$
$q_{i,out,out}$	the flow of outgoing traffic from the node number i in the direction away from the ring
$q_{i,in,out}$	the flow of outgoing traffic from the node number i towards the inside of the ring
$q_{i,out,in}$	the flow of incoming traffic from the node number i in the direction away from the ring
$q_{i,in,in}$	the flow of incoming traffic from the node number i towards the inside of the ring

Simulation realized for the «matrix of correspondence» Q, with the following form:

	0_{in}	0_{out}	$\dots$	$\dots$	$(n-1)_{out}$
0_{in}	$\times$	$Q[0, in, 0, out]$			
0_{out}	$Q[0, out, 0, in]$	$\times$			
$\vdots$			$\times$		
$\vdots$					
$(n-1)_{out}$					$\times$

Table 4. Correspondence matrix for the ring road

To simplify the calculations, the cells are numbered not in the form of a $2n\ times 2n$, but as a $Q[i, in/out, j, in/out]$, i.e., the view of matrix of correspondence has a four-dimensional matrix of size $n\ times 2\ times n\ times 2$. For the numerical notation in the matrix «in» is 0 and «out> is 1. $Q[i, in, n, out]$ refers to the number of vehicles per unit of time, who want to go from the inner of the menor road on i-th node on the outer of the menor road on j-th node.

Let N_i — number of lanes on the corresponding minor roads and N — number of lanes on ring road.

The number of lanes at the entrance supposed sufficient to pass everyone, thus we can use lemma of effective number of lanes. Freeway taken with a constant width, the maximum capacity of one lane — constant equal to S. Was computed maximum capacity of the considered freeway depending on the matrix of correspondence and the capacity of the main and side roads. Previously considered a model of traffic on isolated crossroad generalized and extended to the case. At movement on the previously mentioned scheme expectation of maximum flow-through from the main road on n-th phase is $E[S_i^n] = S \cdot N \cdot \frac{k_i^n}{q_{i-1}}$, from minor roads $E[S_{i,u}^n] = S \cdot N_i \cdot \frac{k_{i,u}^n}{q_{away,i,u}}$. Here $q_{away,i,u} = \sum_{j=0}^{n-1} \sum_{l=0}^{1} Q[j, l, i, u]$ — intensity of the traffic flow wanted to leave freeway on i-th node to the direction $u(in/out)$. k_i^n — total intensity of traffic flow on main road near node i, which can continue their motion on phase n, $k_{i,u}^n$ — total intensity of traffic flow on minor road $u(in/out)$ near node i, which can continue their motion on phase n

For signal-controlled road intersection on one-way road we can use signal cycle with 3 phases.

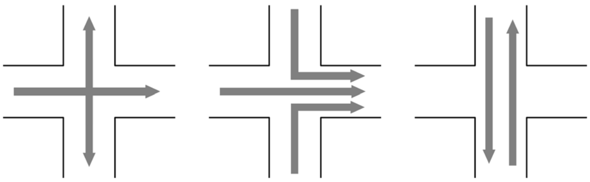

Fig. 4. One of possible scheme of node on one-way ring road

Further we considered traffic lights phases Fig.4.

Let $q_{in,i,u}$ — intensity of incoming traffic flow from minor road u on node i. $q_{in,i,u} = \sum_{j=0}^{n-1} \sum_{l=0}^{1} Q[i, u, j, l]$. Then:

$$c^i \cdot E[q_i] = (c_1^i + c_2^i) \cdot \left(S \cdot N \cdot \frac{q_{i-1} - \sum_{u=0}^{1} q_{away,i,u}}{q_{i-1}} \right) + c_2^i \cdot (S \cdot N_i \cdot \frac{q_{in,i,0} - Q[i,0,i,1]}{q_{in,i,0}} + S \cdot N_i \cdot \frac{q_{in,i,1} - Q[i,1,i,0]}{q_{in,i,1}}), \quad (2)$$

which gives the recurrence relation, which allows from i-th node jump to the $i+1$-th, and it closes the model.

3. A problem to minimize of delays caused by moving on the ring road

«Surplus flows» from each direction for each phase are:

$$P_i^n = \max(q_{i-1} - E[S_i^n], 0),$$
$$P_{i,u}^n = \max(q_{i,u} - E[S_{i,u}^n], 0). \quad (3)$$

The dependence of the P on Q and q_0 can be obtained by substituting obtained values using recurrence receive q_{i-1}.

The delay occurs at the i-th node is equal to:

$$E[W_{sum}^i] = c\Omega_{sum0_i} + \frac{P_{sum1_i}(c_1^i)^2}{2} + \frac{P_{sum2_i}(c_2^i)^2}{2} + c_2^i P_{sum1_i} c_1^i + \frac{P_{sum3_i}(c_3^i)^2}{2} + c_3(P_{sum1_i} c_1^i + P_{sum2_i} c_2^i). \quad (4)$$

It is important to consider that «surplus flows» depend on the flow of the main road, depending on the signal cycle, and that the queues of vehicles depend on the previously located nodes, and, consequently, from the signal cycle.

The software complex of simulation of movement of vehicles on the ring road based on the formula ((4)) and is designed to determine the optimal signal cycles of traffic lights, located on the ring road.

The input data of the program are coefficients $\Omega_{i,j}$, where $i \in \{1, \ldots, N\}$ and $j \in \{In, Out\}$, and $Q_{i,j,k,l}$ where $i, k \in \{1, \ldots, N\}$; $j, l \in \{in, out\}$ and $q_{0i}, i \in \{1, \ldots, N\}$.

The first running mode is to find the optimal solution in assumption that all the lights on the ring road have the similar phase distribution. The solution is coefficients vector (c_1, c_2, c_3), with related condition $c_1 + c_2 + c_3 = 1$, with given values of coefficients $\Omega_{i,j}$, $Q_{i,j,k,l}$ and q_{0i}, which minimises the delay function (4). This problem solves by calculations on grid of possible values c_1, c_2, c_3 with given delta.

The second running mode is to find the optimal solution with each of ring road lights have virtually different distribution of the phases. The solution is vectors set $(c_{1,i}, c_{2,i}, c_{3,i}, i \in \{1, \ldots, N\})$ with the similar constraints, which minimises the delay function (4). This problem is to optimise the function (4) with $2 \cdot N$ variables (the variable $c_{3,i}$ depends of the other variables by expression $c_{3,i} = 1 - c_{1,i} - c_{2,i}$).

4. Conclusion

The paper describes Treiber's model extension by addidion smart agent to the model, providing multivariate behavior, characteristic for traffic on multilane highways and intersections, including complex form, as well as models of circular motion on the ring roads. The model has been tested in specific intersections, testing results are published in a number of works, including [1, 2].

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