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Novel Approach to Modelling Elastic Wavefields in Fractured Media

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SUMMARY

The problem of numerical modelling elastic wavefields in fractured media plays an important role in interpretation of seismic data. The classical approach is based on the use of the averaged models with elastic parameters depending on the internal structure of the medium. Unfortunately, this approach is limited to the predefined types of periodical structures and it cannot describe the crack-crack interactions. Recently, a new approach based on the direct solution of acoustic equations inside the crack was proposed. However, the main drawback of this method is the enormous computational required even with the use of the modern HPC systems. In this paper, we present a novel approach based on numerical solution of the elastic equations with physically correct conditions on the boundaries of the cracks. This approach allows us to describe randomly oriented thin cracks without a drastic increase of computational time. We have demonstrated that Krauklis waves can be achieved by this approach as well. The results of accurate direct simulation of seismic wavefields in heterogeneous fractured media can be used for improved analysis of the seismic data.

Introduction

The problem of numerical modelling elastic wavefields in fractured media plays an important role in interpretation of seismic data. An effective numerical modelling is a critical part of full-waveform inversion algorithms. The classical approach to modelling of elastic waves in fractured geological formation was introduced in papers by Hudson (1981) and Schoenberg *et al*. (1988). It is based on averaging of elastic properties of complex heterogeneous medium taking into account elastic and geometrical properties of inclusions. This approach powers very effective, but, also, it has some limitations; for example, this approach can be used for a limited type of periodical structures only, and it can't describe the seismic waves generated by crack-crack interaction.

Recently, a new approach based on finite-element method was introduced (Frehner *et al*. 2010) that takes into account the internal fluid behaviour inside geological fractions. One of the interesting results is the generation of Krauklis waves (Frehner 2013) at fluid-medium interfaces that may contain important information about the properties of the reservoir rocks. Unfortunately, even with the use of modern HPC systems, the real-scale problem of elastic wave propagation in fractured medium cannot be solved within reasonable time.

In this paper we present a novel approach to modelling elastic wavefields in fractured medium based on applying physically correct boundary conditions in all heterogeneities. First, we show that the same effect as in paper by Frehner (2010) can be obtained by grid-characteristic method, and, second, we present a comparison between the seismic signals obtained with and without our new approach.

Mathematical model and numerical method

The propagation of elastic waves in solid deformable media can be described in the framework of the linear elasticity theory. The governing equations consist of Hooke's law, describing stress-strain relationships, and movement equations, describing stress-acceleration relationships. In this paper, we formulate these equations in terms of components of symmetric stress Cauchy tensor and velocity components. We discuss a two-dimensional case for simplicity but the same calculations were done for three-dimensional case as well.

The variables include three components of stress tensor $\boldsymbol{\sigma} = (\sigma_{xx}, \sigma_{xy}, \sigma_{yy})$, and two velocity components, $\boldsymbol{\upsilon} = (\upsilon_x, \upsilon_y)$. In this case, hyperbolic system of elastic theory equations can be represented as follows:

$$\begin{aligned}
\frac{\partial \sigma_{xx}}{\partial t} &= (\lambda + 2\mu)\frac{\partial \upsilon_x}{\partial x} + \lambda \frac{\partial \upsilon_y}{\partial y},\\
\frac{\partial \sigma_{yy}}{\partial t} &= \lambda \frac{\partial \upsilon_x}{\partial x} + (\lambda + 2\mu)\frac{\partial \upsilon_y}{\partial y},\\
\frac{\partial \sigma_{xy}}{\partial t} &= \mu \frac{\partial \upsilon_y}{\partial x} + \mu \frac{\partial \upsilon_x}{\partial y},\\
\frac{\partial \upsilon_x}{\partial t} &= \frac{1}{\rho}\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y}\right),\\
\frac{\partial \upsilon_y}{\partial t} &= \frac{1}{\rho}\left(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y}\right).
\end{aligned} \qquad (1)$$

Here λ and μ are the Lame parameters, and ρ is density of the medium.

The propagation of acoustic waves in acoustic medium (fluid) can be described in the framework of the acoustic theory. The governing equations can be obtained from system (1) by assuming $\mu = 0$. Our variables include pressure, p, and two velocity components, $\boldsymbol{\upsilon} = (\upsilon_x, \upsilon_y)$. In this case, the hyperbolic system of acoustic equations can be represented as follows:

$$\frac{\partial p}{\partial t} = \lambda\left(\frac{\partial \upsilon_x}{\partial x} + \frac{\partial \upsilon_y}{\partial y}\right),$$
$$\frac{\partial \upsilon_x}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial x}, \qquad (2)$$
$$\frac{\partial \upsilon_y}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial y}.$$

Both of the systems (1) and (2) can be transformed in the same common form equation:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{A_x}\frac{\partial \mathbf{u}}{\partial x} + \mathbf{A_y}\frac{\partial \mathbf{u}}{\partial y} = 0, \quad (3)$$

where $\mathbf{A_x}$ and $\mathbf{A_y}$ are squared matrixes (5×5 and 3×3 for elastic and acoustic equations, respectively), vector $\mathbf{u}$ includes all of corresponding variables. The eigenvalues for any matrix $\mathbf{A}$ from system (3) can be calculated analytically. If we introduce new notations, $c_p = \sqrt{\frac{\lambda+2\mu}{\rho}}$, $c_s = \sqrt{\frac{\mu}{\rho}}$, and $c = \sqrt{\frac{\lambda}{\rho}}$, then the eigenvalues for elastic system will be equal to the following:

$$\lambda_1 = -c_p, \lambda_2 = -c_s, \lambda_3 = 0, \lambda_4 = c_s, \lambda_5 = c_p, \quad (4)$$

and for acoustic system we will have:

$$\lambda_1 = -c, \lambda_2 = 0, \lambda_3 = c,. \quad (5)$$

In this paper, we use different numerical methods to solve system of equations (3). They include finite-volume the TVD method (LeVeque 2004) and the grid-characteristic method (Kholodov *et al.* 2006). By splitting along the spatial directions, system (3) can be transformed into two independent one-dimensional systems. The initial system is transformed into diagonal form by introducing new variables, $\mathbf{A} = \mathbf{R\Lambda R^{-1}}, \boldsymbol{\omega} = \mathbf{R^{-1}u}$. So, the final system can be represented as follows:

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \boldsymbol{\Lambda}\frac{\partial \boldsymbol{\omega}}{\partial x} = 0. \quad (6)$$

All independent transport equations from system (6) are solved using the corresponding numerical scheme. In a general case, when matrix $\mathbf{A}$ depends on coordinates, the generalized numerical methods are used (LeVeque 2004; Kholodov *et al.* 2006).

It should be noted that, for the case of fluid-filled crack we solve system (1) outside of the crack, and system (2) inside the crack, respectively. In this case, the correct values in the nodes at the contact of fluid and medium should satisfy the matching conditions,

$$\boldsymbol{\sigma}\cdot\mathbf{n} = -p, \quad (7)$$

where $\mathbf{n}$ is the normal vector to the crack plane. One of the technical issues is related to the computational complexity of real geological problems. In order to describe accurately the wave processes within heterogeneous medium under consideration, it is necessary to have a fine mesh inside the smallest crack. At the same time, there is a large difference between the crack thickness (~ 1 mm or smaller) and length (~ 1 cm – 100 m), and the modelling domain should have a very large size (e.g., a cube with a side of 10 km).

In order to overcome these difficulties, we introduce a novel approach based on setting physically correct boundary conditions at all heterogeneities representing the fractures. In this case we should have enough (approximately ten) nodes per length of the smallest crack in order to accurately represent the fractures. At the stage of grid generation, the nodes at the borders of cracks are identified and marked as "special". The elastic wavefields are calculated by solving the system of elastic wave equations (1). In the process of solution, we correct the values of the variables in the "special" nodes on every time step to ensure that the linear glide condition is valid, as follows:

$$\boldsymbol{\upsilon}_\mathbf{a}\cdot\mathbf{n} = \boldsymbol{\upsilon}_\mathbf{b}\cdot\mathbf{n}, f_n^{\,a} = -f_n^{\,b}, \mathbf{f}_\boldsymbol{\tau}^\mathbf{a} = \mathbf{f}_\boldsymbol{\tau}^\mathbf{b} = 0. \quad (8)$$

Here $\boldsymbol{\upsilon}$ is the velocity at the node, f_n is the normal component of the interaction force, $\mathbf{f}_{\boldsymbol{\tau}}$ is tangential component of the interaction force, and indexes "a" and "b" denote the different materials.

Examples

We have developed research software based on the described approach. The computational algorithm was parallelized with MPI technology to achieve reasonable performance (see Figure 1). The problem of P-wave interaction with single fluid-filled sub vertical crack was solved. The ratio of the wave length to the crack length is 1/3 and ratio of the thickness to the length is 1/100. Two different cases were investigated: a direct solution of the acoustic equations inside the crack and the use of gliding condition at its borders. The calculation time for the second approach was approximately 10 times less than for the first one. One can see in Figure 2 the beginning of the P-wave interaction with the top crack corner. Figure 3 clearly shows the presence of Krauklis waves.

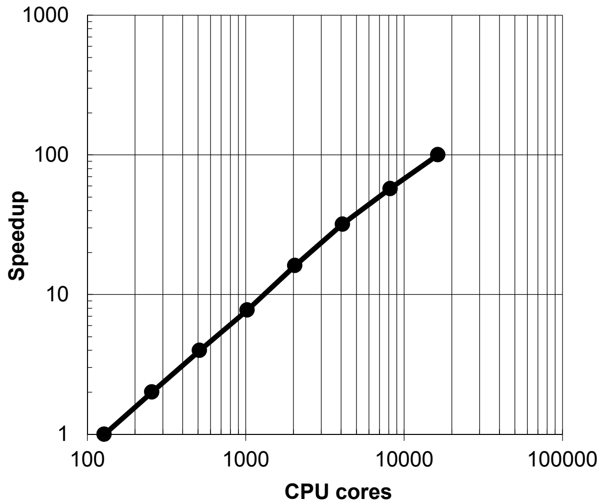

Figure 1 *Scalability test for the developed research software.*

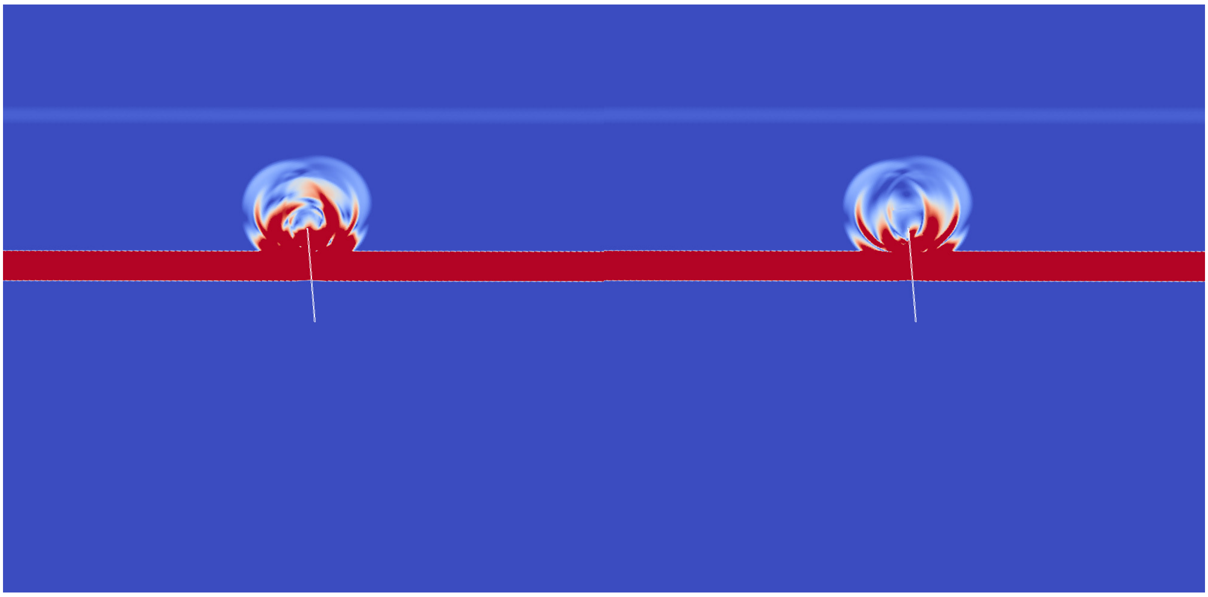

Figure 2 *The images of seismic waves at the beginning of crack - P-wave interaction. We present the acoustic solution on the left and the solution based on the physically correct boundary conditions on the right.*

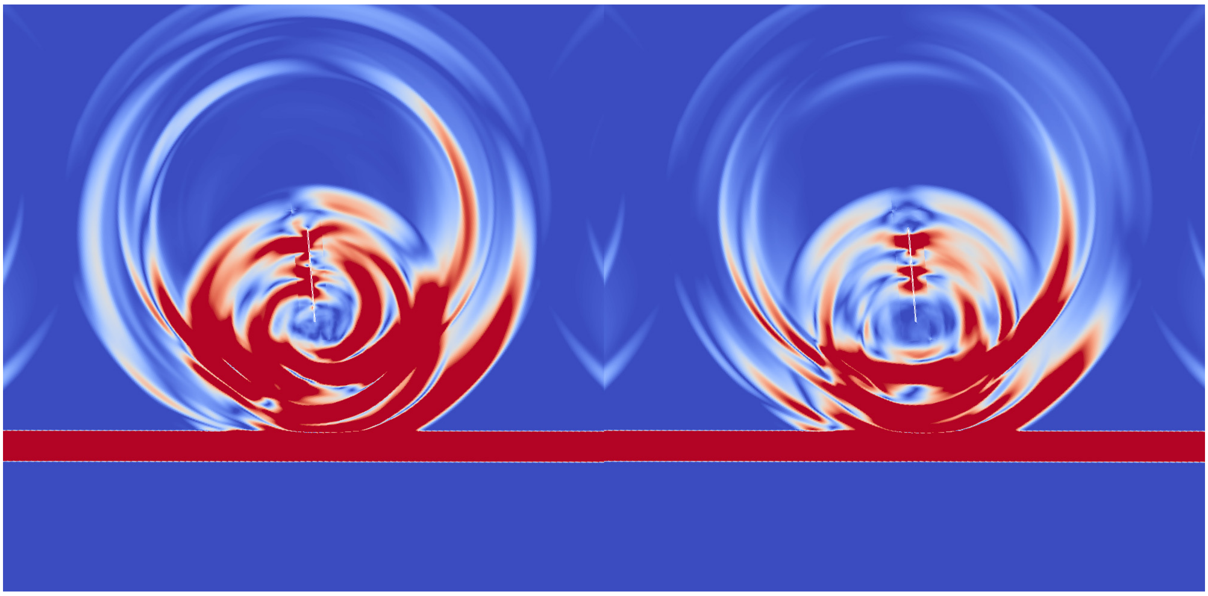

Figure 3 *The images of the seismic waves after the Krauklis wave initiation. We present the acoustic solution on the left and the solution based on the physically correct boundary conditions on the right.*

Conclusions

We present the novel numerical approach for seismic modelling. It is based on the numerical solution of the system of elastic equations with the use of physically correct conditions on the boundaries of geological cracks. This approach improves the accuracy of simulated seismic images while significantly decreasing computation time. The results of accurate direct simulation of the seismic wavefields in heterogeneous fractured media can be used for improved analysis of the seismic data.

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