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Numerical simulation of fracturing in geological medium

Alena Favorskaya[a,b,]*, Igor Petrov[a,b], Anton Grinevskiy[c]

[a]*Moscow Institute of Physics and Technology, 9 Institytsky Pereylok st., Dolgoprudny, Moscow Region, 141700 Russian Federation*
[b] *Scientific Research Institute for System Studies of the Russian Academy of Sciences, 36(1) Nahimovskij av., Moscow, 117218 Russian Federation*
[c] *Lukoil-Engineering, Pokrovskiy blvd.3/1, Moscow, 109028 Russian Federation*

Abstract

Fracturing is the fundamental quality of real rocks, their most important characteristic, the subject of research and the source of information about the geological environment and oil and gas reservoir properties. Models and numerical methods for modeling fracturing zones in geological media, 3 ways to take into account the anisotropy of the geological environments, the model of infinity thin fractures and the conditions for its applicability, 5 types of computational meshes and their advantages in modeling wave processes into fracturing zones, 12 ways of explicitly revealing the structure of a fracturing zone and their differences from each other are discussed, structured and investigated in this paper. Also an example of fracturing modeling using grid-characteristic method and a model of infinity thin fractures is considered. The effect of the source frequency on the seismic responses from fracturing zones is analyzed.

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Keywords: fracturing modeling; cracks modeling; anisotropy; seismic exploration; seismic prospecting; oil exploration; gas exploration; wave processes; numerical modeling; grid-characteristic method

1. Introduction

Nowadays oil and gas industry is one of the main factors forming economics and budget of countries. In recent years, dense fractured rocks and fracturing zones are increasingly involved into oil and gas prospecting and oil and gas exploration. Mapping and parameterization of fracturing zones and pseudo-fractured reservoirs' characteristic

* Corresponding author. Tel.: +7-495-408-66-95; fax: +7-495-408–42–54.
E-mail address: aleanera@yandex.ru

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for these rocks is a very difficult task for a traditional seismic survey. A full solution of this important task has not yet been found. This stimulates the development and subsequent application of new research methods that take into account the physical properties of real. Creation of an information base for their development requires solving direct problems for fractured media. It can be provided only by numerical computational modeling using computers and high performance computer systems.

Thus, fracturing modeling is a crucial problem for geology, seismic prospecting and seismic exploration[1-7]. Nowadays, new numerical approaches for fracturing modeling are being actively developed[8-11]. In the researches[12] special attention is paid to the development of meshes. One can find examples of using grid-characteristic method[13-20] for fracturing modeling in the papers[1-4].

The methods for modeling fracturing zones in geological media, 3 ways to take into account the anisotropy of the geological environments, the model of infinity thin fractures and the conditions for its applicability, 5 types of computational meshes and their advantages in modeling wave processes into fracturing zones, 12 ways of explicitly revealing the structure of a fracturing zone and their differences from each other are discussed in Section 2. An example of fracturing modeling using grid-characteristic method is considered in Section 3. Section 4 contains conclusions.

2. The ways for fracturing modeling

There are two approaches for the modeling of wave processes in fracturing zones of geological media.

- Averaging
- Explicit revealing of the structure of the fracturing zone

The averaging is based on a homogeneous anisotropic or isotropic model of the fracturing zone. The used parameters of the homogeneous medium are calculated on the basis of available information of the fracturing zone under consideration. The averaged model can be one of the following types.

- Isotropic acoustic medium
- Isotropic elastic medium
- Anisotropic elastic medium

The pseudospectral method can be used for seismic elastic isotropic[21] waves propagation modeling[22-24], as well as the method of spectral elements[25-27]. Also grid-characteristic method[13-20] is applied successfully. Anisotropic elastic media is discussed in the researchers[15].

In the case of isotropic acoustic medium, the following system of equations needs to be solved:

$$\rho \partial_t \mathbf{v}(\mathbf{r},t) = -p(\mathbf{r},t) \tag{1}$$

$$\partial_t p(\mathbf{r},t) = -\rho c^2 \left(\nabla \cdot \mathbf{v}(\mathbf{r},t)\right) \tag{2}$$

where $\mathbf{r}$ is a radius vector, t is time, $\mathbf{v}(\mathbf{r},t)$ is a velocity into the isotropic acoustic medium, $p(\mathbf{r},t)$ is a pressure into the isotropic acoustic medium, ρ is a density of the isotropic acoustic medium, c is a speed of sound into the isotropic acoustic medium.

In the case of isotropic elastic medium, the following system of equations solves:

$$\rho \partial_t \mathbf{v}(\mathbf{r},t) = \left(\nabla \cdot \boldsymbol{\sigma}(\mathbf{r},t)\right)^{\mathrm{T}} \tag{3}$$

$$\partial_t \boldsymbol{\sigma}(\mathbf{r},t) = \rho\left(c_p^2 - 2c_s^2\right)\left(\nabla \cdot \mathbf{v}(\mathbf{r},t)\right)\mathbf{I} + \rho c_s^2\left(\nabla \otimes \mathbf{v}(\mathbf{r},t) + \left(\nabla \otimes \mathbf{v}(\mathbf{r},t)\right)^{\mathrm{T}}\right) \tag{4}$$

where $\mathbf{r}$ is a radius vector, t is time, $\mathbf{v}(\mathbf{r},t)$ is a velocity into the isotropic elastic medium, $\boldsymbol{\sigma}(\mathbf{r},t)$ is a stress tensor of rank 2 in the isotropic elastic medium, ρ is a density of the isotropic elastic medium, c_p is a speed of longitudinal waves (P-waves) into the isotropic elastic medium, and c_s is a speed of transverse waves (S-waves) into the isotropic elastic medium.

In the case of anisotropic elastic medium, the system of equations needs to be solved as follows:

$$\rho \partial_t v_i(\mathbf{r},t) = \sum_j \partial_j \sigma_{ij}(\mathbf{r},t) \tag{5}$$

$$\partial_t \sigma_{ij}(\mathbf{r},t) = \sum_k \sum_l C_{ij,kl}\left(\partial_k v_l(\mathbf{r},t) + \partial_l v_k(\mathbf{r},t)\right) \tag{6}$$

where $\mathbf{r}$ is a radius vector, t is time, $v_i(\mathbf{r},t)$ are components of a velocity into the anisotropic elastic medium, $\sigma_{ij}(\mathbf{r},t)$ are components of a stress tensor of rank 2 in the anisotropic elastic medium, ρ is a density of the anisotropic elastic medium, $C_{ij,kl}$ are components of tensor of elastic parameters of rank 4. The components of the tensor $\mathbf{C}$ of rank to satisfy the Eq. 7:

$$C_{ij,kl} = c_{i,k}\delta_{ij}\delta_{kl} + \sum_{m=1}^{3} c_{i,m+3}\delta_{ij}\left|\varepsilon_{mkl}\right| + \sum_{m=1}^{3} c_{m+3,k}\left|\varepsilon_{mij}\right|\delta_{kl} + \sum_{m=1}^{3}\sum_{n=1}^{3} c_{m+3,n+3}\left|\varepsilon_{mij}\right|\left|\varepsilon_{nkl}\right| \tag{7}$$

where δ_{ij} are components of unit tensor of rank 2, ε_{mkl} are components of absolutely ant non-symmetric unit tensor of rank 3, c_{ik} are uncial coefficients characterizing the anisotropic linear elastic medium. These uncial coefficients c_{ik} are typically placed into the following structure in a matrix view.

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{pmatrix} \tag{8}$$

Notice that the structure in Eq. 8 is not a matrix and is not a tensor of rank 2.

Usually the case of transversal anisotropy is considered to model the fracturing. In the case of transversal anisotropy with the emphasized axis OZ, the structure in Eq. 8 takes the following form.

$$\begin{pmatrix} c_{11} & c_{11} - 2c_{66} & c_{13} & 0 & 0 & 0 \\ c_{11} - 2c_{66} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix} \tag{9}$$

Notice that when the emphasized axis of transversal anisotropy does not correspond with one of the coordinate axes the structure in Eq. 9 returns to the arbitrary case in Eq. 8.

According to the investigation[1-4], the approach of averaging does not allow to take into account all types of seismic waves reflected from the fracturing zone. This conclusion is confirmed not only by the numerical but also by the field experiments. At the same time, the averaging approach makes it possible to substantially simplify the computational complexity of the problem.

The explicit revealing of the structure of the fracturing zone allows to obtain all types of seismic waves reflected from the fracturing zones that exist in nature.

Thus, one can distinguish the following three ways of taking into account the anisotropy of geological environments.

- Model of an anisotropic elastic medium
- Model of an isotropic elastic or acoustic medium with explicit revealing of the structure of fracturing zones
- Model of an anisotropic elastic medium with explicit revealing of the structure of fracturing zones

It is also possible to single out a whole family of ways for the explicit revealing of the structure of fracturing zones. One can find several illustrations in Fig. 1.

First, consider the ways to account of each fracture in the fracturing zone under consideration.

- Explicit revealing of each inhomogeneity. See examples in Fig. 1a, 1b
- Model of infinity thin fractures. See examples in Fig. 1c, 1d, 1e, 1f

Explicit revealing of each of the inhomogeneity more accurately describes the wave processes in the fracturing zone. A model of infinitely thin fractures allows to save computing resources.

According to the criterion considered in[4], if the ratio of the aperture of the fracture to the length of the fracture is greater than 1:1000, the model of infinitely thin fracture allows to consider all types of reflected from fracturing zone seismic waves. This criterion is not strict. For example, the model of infinitely thin fractures could be applied if the ratio is 1:100.

The infinitely thin fractures can be modeled with different characteristics.

- Fluid-filled infinitely thin fractures
- Gas-filled infinitely thin fractures
- Infinitely thin fractures with a variable parameter

Second, various types of computational meshes can be used for the explicit revealing of the heterogeneous structure of the fracturing zone.

- Regular meshes. See examples in Fig. 1a, 1c
- Curvilinear hexahedral grids. See an example in Fig. 1d
- Unstructured tetrahedral and triangular meshes. See examples in Fig. 1b, 1e, 1f
- Hierarchical meshes. See an example in Fig. 1f
- Mixed meshes

Grid-characteristic method on regular meshes is considered at the papers[13,17]. The curvilinear hexahedral meshes are discussed at the works[3,18]. Finite difference schemes on triangular and tetrahedral grids are investigated in the articles[28-34]. Grid-characteristic method on unstructured triangular and tetrahedral grids is studied in the works[1,2,16,20]. Hierarchical meshes are discussed in the paper[20]. The comparison of grid-characteristic method and other techniques of wave processes modeling are adduced in the work[16].

The advantages of using structural meshes are the minimal complexity of calculation and the minimal expenditures of computing resources, such as Random Access memory (RAM) and time counting. The advantage of using unstructured tetrahedral and triangular grids is a possibility of representation of the explicitly identifying irregularities, boundaries, and contact boundaries of arbitrary shape. The advantages of both approaches are partly present when curvilinear hexahedral meshes are used. Structural, and curvilinear hexahedral, and unstructured tetrahedral and triangular meshes could be hierarchical. Also, all types of meshes can be implemented in one calculation. This implementation of different types of meshes in one calculation is the use of mixed meshes. Using hierarchical and mixed computational meshes allows to reduce the expenditures of RAM and time counting.

We can also distinguish the following two ways of explicitly revealing of the inhomogeneity.

- Explicit revealing of contact boundaries and interfaces. See examples in Fig. 1b, 1d, 1e, 1f
- Though calculations. See examples in Fig. 1a, 1c

The advantage of the explicit revealing of contact boundaries and interfaces is the guarantee to take into account all wave processes being in nature. The advantages of using of the through calculations is to reduce the computational complexity of the problem.

Summarizing one can distinguish 12 types of revealing of the heterogeneous structure of the fracturing zone.

- Regular meshes. Explicit revealing of contact boundaries and interfaces. Explicit revealing of each inhomogeneity
- Regular meshes. Though calculations. Explicit revealing of each inhomogeneity. See an example in Fig. 1a
- Curvilinear hexahedral meshes. Explicit revealing of contact boundaries and interfaces. Explicit revealing of each inhomogeneity
- Curvilinear hexahedral meshes. Though calculations. Explicit revealing of each inhomogeneity
- Unstructured tetrahedral and triangular meshes. Explicit revealing of contact boundaries and interfaces. Explicit revealing of each inhomogeneity. See an example in Fig. 1b
- Unstructured tetrahedral and triangular meshes. Though calculations. Explicit revealing of each inhomogeneity
- Regular meshes. Explicit revealing of contact boundaries and interfaces. Model of infinity thin fractures
- Regular meshes. Though calculations. Model of infinity thin fractures. See an example in Fig. 1c
- Curvilinear hexahedral meshes. Explicit revealing of contact boundaries and interfaces. Model of infinity thin fractures. See an example in Fig. 1d
- Curvilinear hexahedral meshes. Though calculations. Model of infinity thin fractures
- Unstructured tetrahedral and triangular meshes. Explicit revealing of contact boundaries and interfaces. Model of infinity thin fractures. See examples in Fig. 1e, 1f
- Unstructured tetrahedral and triangular meshes. Though calculations. Model of infinity thin fractures

Also, in each of the 12 cases, the grid can be hierarchical and / or mixed. See an example of hierarchical tetrahedral mesh in Fig. 1f.

Grid-characteristic method is successfully applicable and using in the cases of all of fracturing modeling approaches discussed in this Section.

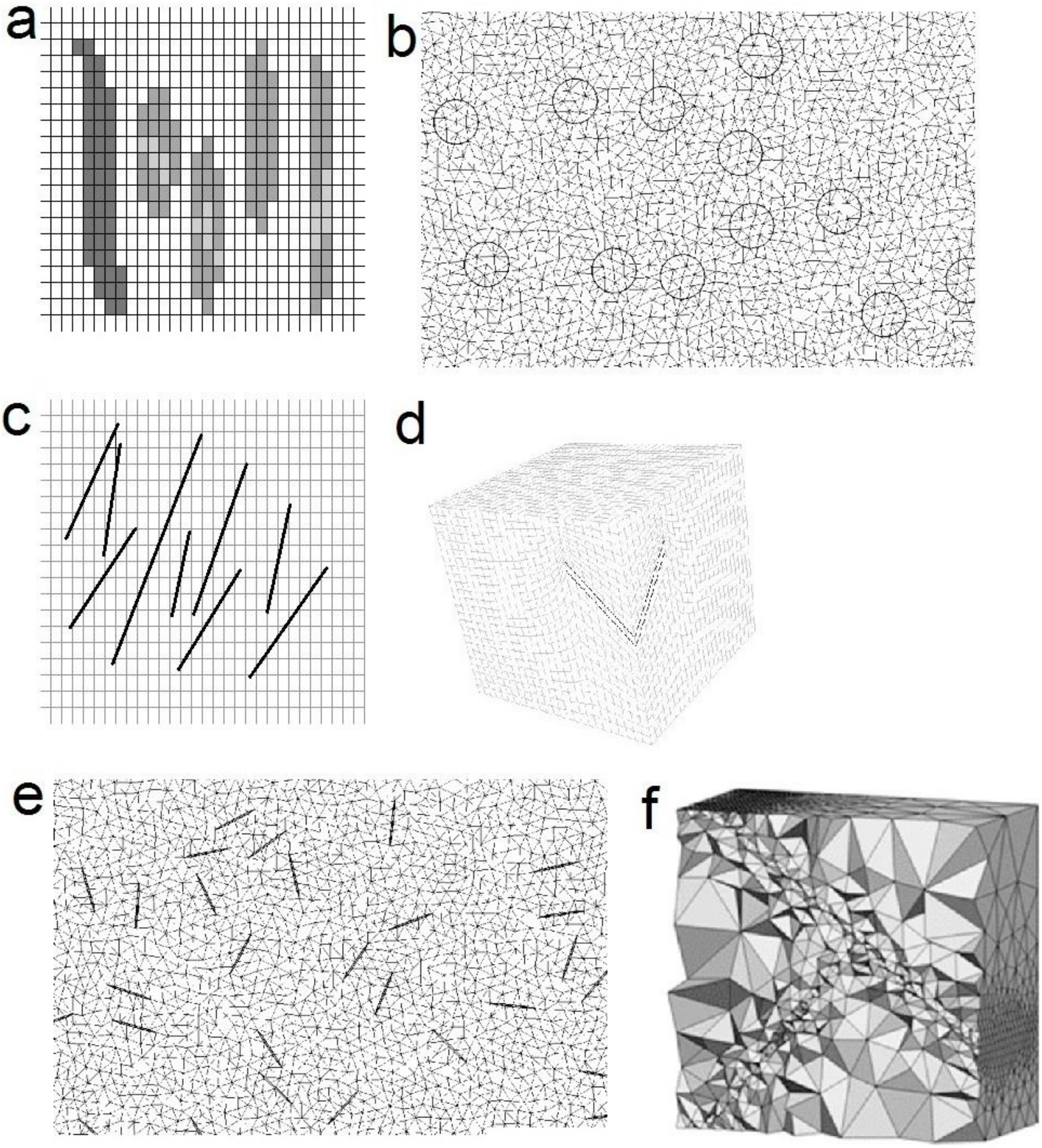

Fig. 1. The ways for the explicit revealing of the structure of fracturing zones.

3. An example of fracturing modeling

The following physical parameters of fracturing zone to be investigated were specified. The dimensions of the cracks are the following values. The apertures are from 0.05 to 0.5 mm. The lengths and widths are from 200 to 5000 times greater than the apertures. The speed of P-waves in the enclosing rocks was equal to 6300 m/s. The speed of S-waves in the enclosing rocks was equal to 3200 m/s. The density of enclosing rocks was equal to 2.78 g/cm^3. The speed of P-waves within the cracks was equal to1500 m/s. The density within the cracks was equal to1 g/cm^3. The distance between cracks was from 0.02 m to 0.2 m. Due to the size of cracks is more then their apertures in 5000 times according to the criterion considered in the paper[4] it is expediently to use the model of infinitely thin fluid-filled fractures. Two values of frequency of the source were considered: 120 Hz and 1200 Hz. Three variants of cracks disposition were considered.

One can see the wave patterns for different frequencies in Fig. 2, 3. One can see P-wave scattering on the fracturing zone under consideration from the source with the frequency 120 Hz in Fig. 2a. One can see S-wave scattering on the fracturing zone under consideration from the source with the frequency 120 Hz in Fig. 3a. One can see P-wave scattering on the fracturing zone under consideration from the source with the frequency 1200 Hz in Fig. 2b. One can see S-wave scattering on the fracturing zone under consideration from the source with the frequency 1200 Hz in Fig. 3b.

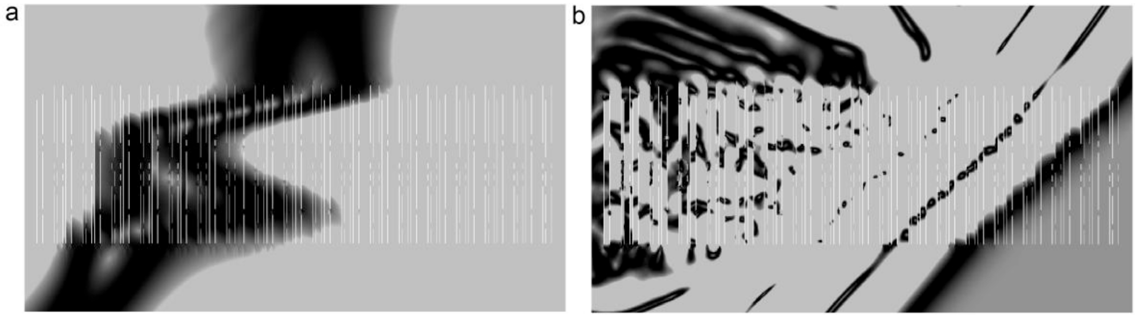

Fig. 2. P-wave scattering into the fracturing zone. The influence of source frequency.

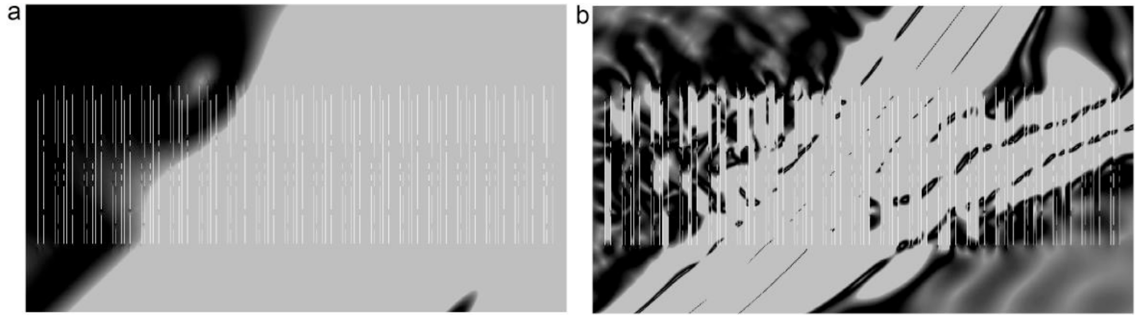

Fig. 3. S-wave scattering into the fracturing zone. The influence of source frequency.

One can see the arising of waves reflected from the different fracturing zones under consideration in Fig. 4, 5 in the case of source frequency 1200 Hz. There is the same time moment in the Fig. 4a, 4b, 4c. And also there is the same time moment in the Fig. 5a, 5b, 5c.

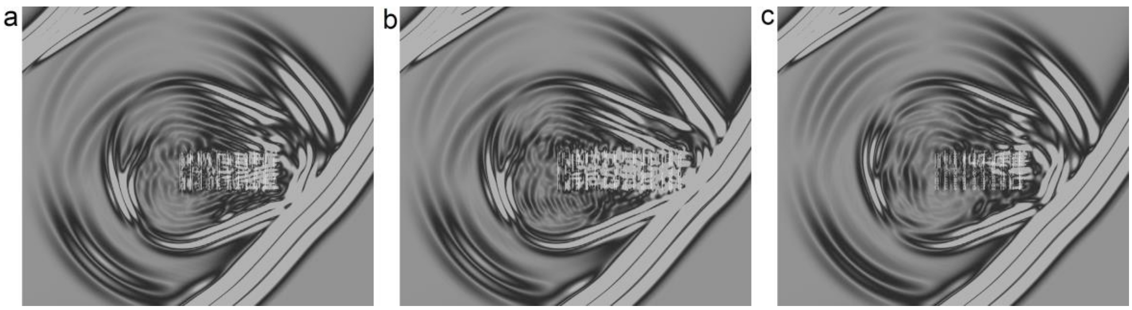

Fig. 4. P-wave reflecting from the fracturing zone. The influence of the structure of the fracturing zone.

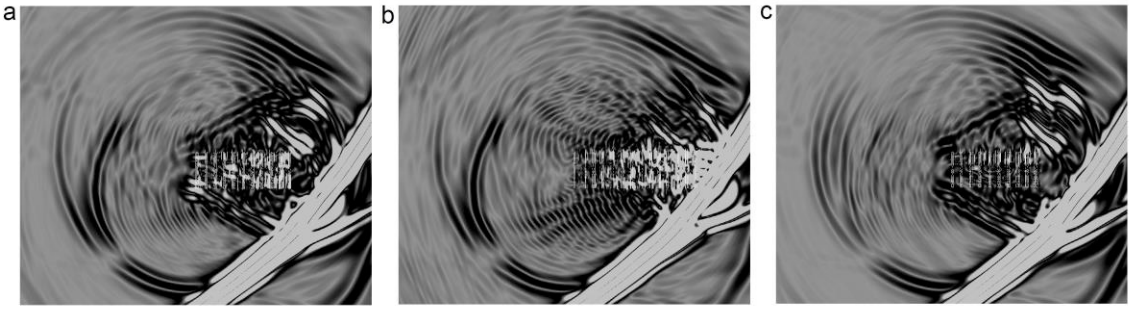

Fig. 5. S-wave reflecting from the fracturing zone. The influence of the structure of the fracturing zone.

4. Conclusions

Models and numerical methods for modeling fracturing zones in geological media, ways to take into account the anisotropy of the geological environments, the model of infinity thin fractures and the conditions for its applicability, types of computational meshes, ways of explicitly revealing the structure of a fracturing zone are discussed, systematized and analyzed in this paper.

The explicit revealing of the structure of the fracturing zone allows to obtain all types of seismic waves reflected from the fracturing zones that exist in nature. Explicit revealing of each of the inhomogeneity more accurately describes the wave processes in the fracturing zone. The use of the model of homogeneous anisotropic media, the model of infinitely thin fractures and the thought calculation allows to save computing resources.

If the ratio of the aperture of the fracture to the length of the fracture is greater than 1:1000, the model of infinitely thin fracture allows to consider all types of reflected from fracturing zone seismic waves. This criterion is not strict. For example, the model of infinitely thin fractures could be applied if the ratio is 1:100.

The advantages of using structural meshes are the minimal complexity of calculation and the minimal expenditures of computing resources, such as Random Access memory (RAM) and time counting. The advantage of using unstructured tetrahedral and triangular grids is a possibility of representation of the explicitly identifying irregularities, boundaries, and contact boundaries of arbitrary shape. The advantages of both approaches are partly present when curvilinear hexahedral meshes are used. Using hierarchical and mixed computational meshes allows to reduce the expenditures of RAM and time counting.

Grid-characteristic method is successfully applicable and using in the cases of all of fracturing modeling approaches discussed in the Section 2.

Also an example of fracturing modeling using grid-characteristic method is considered. The effect of the source frequency on the seismic responses from fracturing zones is analyzed. The advantages of using a source with a higher frequency of seismic waves to determine the characteristics of a fractured zone under investigation have been identified and illustrated by dynamical wave processes. Grid-characteristic method using infinity thin fracturing allows to take into account the anisotropy in the seismic response for different frequencies of sources. Higher frequency of source allows to investigate more properties of fracturing zone.

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