International Conference on Knowledge Based and Intelligent Information and Engineering Systems, KES2017, 6-8 September 2017, Marseille, France

Numerical modeling of ultrasound beam forming in elastic medium

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Abstract

Modeling of ultrasonic pulse in a medium is usually based on the system of equations of acoustics. This approximation means that, when a wave pattern is considered, only longitudinal waves are taken into account. This approach works well in many practically meaningful applications. However, for some cases the consideration of only longitudinal waves is not enough, and an analysis of the complete elastic wave pattern is required. This paper is devoted to the modeling of an ultrasonic pulse in medium using the system of equations of elasticity instead of the system of equations of acoustics. A transducer with phased array is considered. Grid-characteristic numerical method is used for a numerical solution.

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Peer-review under responsibility of KES International

Keywords: numerical modelling; grid-characteristic method; ultrasound; phased array; non-destructive testing

1. Introduction

Ultrasound techniques are among the most common methods of diagnostics widely used in many areas: from non-destructive testing of parts in industry to medical studies. One of the basic assumptions, on which ultrasound studies are based, is the assumption of the uniformity of the sound velocity in the object under study. Thus, modeling of ultrasound pulses is usually based on the system of equations of acoustics. For many cases, this is completely justified. For example, studies show that in soft biological tissues the attenuation factor for shear waves is four orders of magnitude higher than for longitudinal ones\textsuperscript{1,2}. Thus, when modeling ultrasound in such medium, it is
correct to limit wave pattern to the consideration of longitudinal waves only, describing their propagation by the system of acoustic equations and using the Maxwell model for the damping simulation\(^3\).

However, for some objects the system of equations of acoustics is not enough. In most industrial materials, the shear waves do not fade so rapidly and the ultrasound wave pattern is much more complicated – one can observe both longitudinal and transverse waves, as well as Rayleigh and Lamb waves. This becomes extremely important, when the acoustically contrasting objects are studied.

For example, a scanning of soft tissues of the brain through the skull bones\(^4\) – bone tissue has a complex shape and rheological parameters that differ significantly from the parameters of soft tissues. This leads to the fact that the skull distorts the wave fronts, which propagate from ultrasound transducer and back. Even small irregularities of the skull caused by individual features can distort the resulting image to full unrecognizability.

At the moment, the only reliable method of brain vessels damage diagnostics is angiography, which requires an insertion of radiopaque agents, and has a lot of contraindications. A reliable method of ultrasound image recovery after a disturbance from the skull will allow diagnosing vascular diseases safely and on an earlier stage. Existing methods\(^5\) of transcranial visualization require a high-resolution brain computed tomography as an input. This is a serious drawback, because it greatly increases the cost of procedure.

Another reason for the elastic material model application is the existence of noise in lateral sectors\(^6\) and a “blind spot” in a small area directly beneath the sensor\(^7\). Both effects are explained by shear waves, which are not considered by an acoustic material model, but exist in real objects. Even in soft tissues shear waves manage to leave an imprint on the signal before disappearing due to the attenuation. A reliable model of shear waves behavior will help to improve visualization methods, reducing the noise.

This paper is a part of a large work, dedicated to the development of a transcranial ultrasound method\(^8\). It is devoted to the numerical simulation of an ultrasonic pulse in medium, using a system of elasticity equations instead of a system of acoustic equations. The formation of a pulse by a transducer with a phased array and its focusing are considered. The aim of the paper is to obtain from the first principles the complete wave pattern in the formation of an impulse. The authors hope that further development of this approach will provide a tool that can facilitate and accelerate the development of new algorithms for processing the ultrasound signal for complex cases, as well as the deeper understanding of processes, occurring in the transducer and the object under study.

The remainder of this paper is organized as follows. Section 2 describes the structure and operation principles of ultrasound phased arrays. The model and numerical method are briefly described in Section 3. Section 4 presents the results of numerical modeling, and the major results are discussed in Section 5.

2. Structure of transducers with phased array

Typical ultrasonic pulse consists of several wave periods and contains a frequency spectrum determined by the central frequency and the width of the distribution (Fig. 1). The high frequency transducer generates rather short pulses (1-3 wave periods). These pulses have a wide spectrum, which can be seen on their Fourier transforms. This transducer allows to obtain accurate information about distances with good axial resolution and is ideal for a two-dimensional diagnostic imaging. The low frequency transducer generates long pulses (5-30 wave periods), having a narrow spectrum, respectively. It allows to receive accurate information about the frequencies in the medium but a poor axial resolution, which is suitable for the Doppler method.

A short pulse after reflection from the obstacle generates a single reflection, which allows to measure the time (and the distance under assumption of the uniformity of the sound velocity) to the obstacle. A longer pulse makes it difficult to identify individual reflections, since they overlap and introduce significant errors due to their length.

All phased array transducers have a similar structure\(^8\): a thin piezoelectric plate (usually lead zirconate titanate), a matching layer, an insulating layer, and one lens, covering the array of sensors. The piezoelectric plate generates and receives ultrasonic pulses. The crystal is in the receiver state for the most of the time (about 99%).

The main disadvantage of lead zirconate titanate is its high acoustic density in comparison with the most of materials considered. This leads to the fact that about 80% of the energy of the ultrasonic wave is reflected from the material boundary and a multiple internal reflection occurs in piezoelectric plate itself. To reduce the effect of reflections, an insulating layer is placed on the back side of the piezoelectric plate. This layer damps most of the waves that pass into it.
To smooth the effect from the acoustic contrast between the materials, a matching layer with intermediate acoustic properties is placed in front of the piezoelectric plate. It allows to increase the fraction of passing energy to almost 100%. To achieve this, its thickness should be exactly one quarter of the length of the ultrasonic wave. This significantly reduces the spectrum, which harms the resolution, when a short pulse is used. To extend the spectrum, a combination of several connecting layers with different properties and different widths is applied. A curved cylindrical lens is used to focus the beam in a vertical plane and makes it easier to focus electronically increasing the amplitude of the signal in the focal zone. The different arrangement of the phased array elements allows the beam to be focused in different planes.

![Image](image_url)

**Fig. 1.** (a) short pulse, (b) long pulse.

Transducers with phased array for processing 2D images usually have 128 rectangular elements (arrays), each of which transmit and receive a signal separately. Transducers for working with three-dimensional images have more elements, up to 3,000. All elements are used to receive and transmit signals on each scan direction, and each scan direction matches the axis of the beam. The signal frequency decreases as the size of a single element increases. The use of phased arrays permits to control the focusing and beam direction without moving the sensor by changing the frequency and phase of the ultrasonic wave transmission in each element. Accordingly, after receiving the signal for its analysis the appropriate processing is necessary.

The direction of the beam (the resulting elastic wave) can be controlled using the Huygens-Fresnel principle. Each element of the wave front can be considered as the source of secondary spherical waves and the resulting field at each point of space will be determined by the interference of these waves. If the phased array elements emit a wave with the same amplitude and wavelength, their linear size is larger than the wavelength, and the phases of their signals are matched, then an elastic wave with a direct front propagating at an angle to the surface will be obtained as a result of the interference of these waves.

Thus, the entire scope can be scanned without changing the sensor position. Automatic scanning reduces the human factor simplifying both signal processing and data interpretation. The scanning is usually performed using one of two algorithms: the sector scan or linear scan. The size of the object being studied is determined by the difference in the arrival time of the signal from the top and bottom of the object. With sector scanning, an appropriate correction must be made but the area under investigation can be substantially wider than the sensor size. Similarly, by controlling the phases of individual elements, the shape of the front can be changed – particularly focused at the desired point.

It is also necessary to note the problem that arises during the scanning at large angles – the "noise" that occurs in the lateral sectors. Each of the elements, when interacting with the target elastic material, generates a complex wave pattern, from which only the longitudinal wave of the spherical shape is subsequently analyzed. In addition to the longitudinal elastic waves, the transverse and surface waves are also generated propagating along the sensor with other velocities and form and contribute in the signal on the elements. However, the conventional algorithms are limited to a uniform sound velocity assumption and this contribution is interpreted as a noise. This noise has high amplitude and significantly reduces the resolution in the near zone and the lateral sectors.
3. Transducer model and numerical method

In this paper, a rectangular two-dimensional flat sensor Sonomed-500 is modeled. The size of the element is approximately equal to \( \lambda/2 \). Since this sensor is designed for frequencies of 2.5-3.5 MHz, the cell size is approximately 0.22 mm. The phase of the source is modeled by the boundary condition. The sensor sizes are \( 1 \times 1 \) cm\(^2\). The phase of the receiver will be modeled by measuring the pressure on the surface at the points, where the boundary condition was specified. Also, the normal speed at these points is measured.

The receiving starts immediately after transmitting the pulse and lasts \( 2z_{\text{max}}/c \) seconds, where \( c \) is the speed of sound, \( z_{\text{max}} \) is the maximum scanning depth that is expected to be obtained in a particular measurement. The beam is focused by introducing the corresponding delays on the elements of the array. After the triggering of each element, a complex wave pattern is formed in the object under investigation. The major wave is a spherical longitudinal one. When these waves from different elements are interfered, a single wave front is formed, the shape of which can be changed due to a different delay, when elements are switched on. Focusing this front at a certain point maximizes the response amplitude from inhomogeneity at this point and simplifies their recognition. If the sensor is two-dimensional, the focusing can be carried out in both directions.

The modeling for deformable solid based on the system of Eqs. 1 consists of motion equations, rheological correlations, and the equation of state:

\[
\begin{align*}
\rho \dot{v}_i &= \nabla_{\!j} \sigma_{ij} + f_i, \\
\sigma_{ij} &= q_{ijkl} \varepsilon_{kl} + F_{ij},
\end{align*}
\]

where \( \rho \) is a density, \( v_i \) is a displacement speed components, \( \nabla_{\!j} \) is a covariant derivative by \( j \) coordinate, \( \sigma_{ij} \) is a stress tensor, \( \varepsilon_{ij} \) is a strain tensor, \( f_i \) is a mass forces, \( F_{ij} \) is the right part, depending on a rheology model, \( q_{ijkl} \) is the fourth order tensor also depending on a rheology model.

It is required to solve the set of equations the hybrid grid-characteristic method of 1-2 order on irregular tetrahedral grid\(^{11,12,13,14} \). This method is based on the characteristic properties of the elastic deformable solids set of equations into account and models the accurately propagation, reflection, and refraction of wave fronts, including their interconversion on different border and contact types.

4. Numerical results

In this paper, we consider a homogeneous polymer cube (\( \rho = 1041 \ \text{kg/m}^3, \ \text{c}_p = 1595 \ \text{m/s} \)) with the sizes of 10x10x10 cm\(^3\). The phased array is placed on the face of the cube with \( x = -5 \) cm. The opposite side of the cube \( x = 5 \) cm is a free boundary, from which the waves are reflected. The other faces are absorbing boundary conditions. This is equivalent to ultrasonic scanning of a large 10-cm thick plate. All the waves that go beyond the computational domain are absorbed (physically this is equivalent to scattering and attenuation in a large volume). The impulse was modeled as a Gauss function superimposed on the sinus function – the frequency of the sine is \( \omega = 0.3 \) MHz, the width of the Gauss function is \( \tau = 0.2 \ 10^9 \) s.

Firstly, we consider an unfocused pulse. All elements are triggered simultaneously, generating a plain wavefront, and the reflected signal is recorded without delay. The wave pattern is shown in Fig. 2 and Fig. 3. It can be seen that the front of the wave is straight and do not inclined to the face, the entire signal is reflected from the back face. The longitudinal wave pattern is shown in Fig. 2. The reflected diagnostic pulse returns from the backside, reflects from the sensor and returns to the backside. Several re-reflections (multiple signals from the backside) are a common occurrence in cover study\(^7 \). The transversal wave pattern, specific for elastic model, is shown in Fig. 3. Transversal waves are emitted from the sensor borders and propagate to its center. If the inhomogeneity under study (crack or pore in a metal, vessel or tumor in a biological tissue) was located near the sensor, its ultrasound response would have been lost in the interfering transversal waves signal. It makes the area directly adjacent to the sensor effectively unreachable by the ultrasound diagnostics. The transversal waves also propagate to the backside and reflect from it, creating a low-amplitude “shadow” on the resulting image due to their lower speed, compared to the longitudinal waves.
Fig. 2. without focusing, wave propagation over time, v_z component in XZ section
Fig. 3. without focusing, wave propagation over time, $v_i$ component in XZ section
Fig. 4. focusing on the backside (lower point), wave propagation over time, $v_x$ component in XZ section
Fig. 4. focusing on the backside (lower point), wave propagation over time, $v_x$ component in XZ section

Fig. 5. focusing on the backside (higher point), wave propagation over time, $v_x$ component in XZ section
Consider focusing on the backside. A series of calculations was carried out with focusing at different heights. The real sensor typically makes about 200 measurements during ultrasonic study, scanning the angle of about 30°. We limited ourselves to a smaller number of rays for this preliminary calculation. The wave pattern is shown in Fig. 4 and Fig. 5. Sensor elements emit their pulses each at the given time forming a concave wave front, converging to a target point on the backside. The transversal and longitudinal waves are harder to display explicitly in this case. Thus, data to a single velocity component were limited. The noise caused by transversal waves can be seen along the sensor border during the whole measurement. In comparison with the direct wave front, the focused one fades more quickly lowering the amplitude of multiple reflections.

5. Conclusions

In this paper the wave pattern in the medium during ultrasonic pulse forming by a transducer with a phased array is obtained using the system of elasticity equations. Pulse propagation and focusing are considered. The key feature of the results is that they demonstrate the complex wave pattern, appearing due to the consideration of the elastic material model instead of the acoustic one. Specification of the behavior of shear waves can help to assess parameters of lateral noise and “blind area”.

Applying the method to the transcranial ultrasound problem will allow to consider the overall influence of bone layer on a resulting image, providing a tool that can facilitate and accelerate the development of new algorithms for processing the ultrasound signal for complex cases, as well as the deeper understanding of the processes, occurring in the sensor and the object under study.

Acknowledgements

The reported study was funded by the Russian Fund for Basic Researches according to the research project № 16-07-00884 A.

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