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Polarization characteristics of radiation in both 'light' and conventional undulators

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ABSTRACT

As a rule, an intensity spectrum of undulator radiation (UR) is calculated by using the classical approach, even for electron energy higher than 10 GeV. Such a spectrum is determined by an electron trajectory in an undulator while neglecting radiation loss. Using Planck's law, the UR photon spectrum can be calculated from the obtained intensity spectrum, for both linear and nonlinear regimes.

The electron radiation process in a field of strong electromagnetic waves is considered within the quantum electrodynamics framework, using the Compton scattering process or radiation in a 'light' undulator. A comparison was made of the results from using these two approaches, for UR spectra generated by 250-GeV electrons in an undulator with a 11.5-mm period; this comparison shows that they coincide with high accuracy. The characteristics of the collimated UR beam (i.e. spectrum and circular polarization) were simulated while taking into account the discrete process of photon emission along an electron trajectory in both undulator types. Both spectral photon distributions and polarization dependence on photon energy are 'smoothed', in comparison to that expected for a long undulator—the latter of which considers the ILC positron source (ILC Technical Design Report).

1. Introduction

One possible option for a polarized positron source for use with future International Linear Collider (ILC) is a helical undulator, which is needed to produce a beam of circularly polarized photons with the subsequent generation of longitudinally polarized positrons [1].

As for the considered parameters of the polarized positron source, the energy of electrons is about 150–250 GeV, the period of the undulator is $\lambda_u = 11.5$ mm, the undulator strength parameter is $K \sim 1$, and the number of periods is $N_u = 2 \times 10^4$). The energy of the emitted photons, meanwhile, is $\hbar\omega > 10$ MeV. In other studies [2–4], the undulator radiation (UR) characteristics are calculated according to the formulas of classical electrodynamics—which, generally speaking, require justification, as the photons of UR at the indicated energy level are emitted in a discrete manner. In other words, in every act, the energy of electron changes abruptly. The UR process, at the very least, should therefore be described within the framework of quantum theory.

This study describes the UR, based on an analogy with radiation from electrons in a 'light' undulator [5,6]. That is, the sequential simulation of each event corresponding to the emission of the UR photon of radiation is treated as a process of nonlinear Compton scattering.

2. Theory of undulator radiation in the classical approach

To begin with, let us consider classical UR.

For the sake of simplicity, we consider UR from a helical undulator with a period λ_u, an undulator strength parameter K (nonlinearity parameter), and a number of periods N_u. In such an undulator, the electron trajectory is a helix. In a system where an electron is, on average, at rest (R-system), the trajectory of an electron is circular with the radius [7]:

$$R \approx \frac{K\ \lambda_u}{\pi\gamma_0}, \text{ if } \quad K \sim 1, \tag{1}$$

where γ_0 is the Lorentz factor.

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Using the well-known formulas for synchrotron radiation, one can easily obtain formulas for radiation intensity after Lorentz-transformation from the R-system to the laboratory system [7]:

$$\frac{dW}{d\Omega} = \frac{8\alpha\hbar\omega_0 N_u \gamma_0^4}{\left(1+K^2+\gamma_0^2\theta^2\right)^3} K^2 \sum_{n=1}^{\infty} n^2 \left[J_n'^2(nZ) + \left(\frac{\gamma_0\theta}{K} - \frac{1}{Z}\right)^2 J_n^2(nZ)\right], \quad (2)$$

where $\alpha = 1/137$ is the fine structure constant; $\omega_0 = \frac{2\pi c}{\lambda_u}\left(1 - \frac{1+K^2}{2\gamma_0^2}\right)$ is the frequency of the fundamental harmonic; θ is the angle of the outgoing photon, $Z = \frac{2K\gamma_0\theta}{1+K^2+\gamma_0^2\theta}$; $n = 1, 2, 3, \ldots$ is the harmonic number; and $J_n(x)$ and $J\prime_n(x)$ are the Bessel function of order n and its derivative, respectively.

The formula (2) is obtained in the small-angle approximation of the outgoing photon for the long undulator ($N_u \gg 1$). There is the well-known relationship connecting the frequency of the n-th harmonic and the outgoing angle [7]:

$$\omega^{(n)} = n \frac{2\gamma_0^2}{1 + (\gamma_0\theta)^2 + K^2} \frac{2\pi c}{\lambda_u}. \quad (3)$$

Using this relationship, it is convenient to transform expression (2) into the spectral distribution ($dW/d\Omega \to dW/d\omega$) and then use a dimensionless spectral variable $S^{(n)}$ instead of a frequency $\omega^{(n)}$:

$$S^{(n)} = \frac{\omega^{(n)}}{2\gamma_0^2\omega_0} = \frac{\omega^{(n)}}{2\gamma_0^2 2\pi c/\lambda_u} = \frac{n}{1 + (\gamma_0\theta)^2 + K^2}, \quad (4)$$
$$0 \leqslant S^{(n)} \leqslant S_{max}^{(n)}, \quad S_{max}^{(n)} = \frac{n}{1+K^2}.$$

The photon UR spectrum for the n-th harmonic can be obtained from the UR intensity spectrum divided by the emitted photon energy $\hbar\omega^{(n)}$:

$$\frac{dN^{(n)}}{dS^{(n)}} = 4\pi\alpha K^2 N_u \left\{ \frac{\left[n - S^{(n)} 2\left(1+K^2\right)\right]^2}{4S^{(n)} K^2 \left[n - S^{(n)}\left(1+K^2\right)\right]} J_n^2(nZ) + J_n'^2(nZ) \right\}. \quad (5)$$

The number of photons emitted at the n-th harmonic can be calculated by integration of the spectral distribution (5):

$$N^{(n)} = \int_0^{S_{max}^{(n)}} \frac{dN^{(n)}}{dS^{(n)}} dS^{(n)}. \quad (6)$$

The total number of photons N_{tot} per electron in the undulator is:

$$N_{tot} \approx \frac{2}{3}\pi\alpha K^2 N_u. \quad (7)$$

For $N_u \sim 10^4$ and for $K \sim 1, N_{tot}$ is approximately equal to 100.

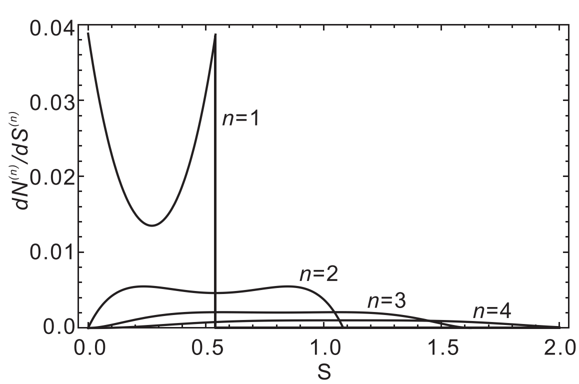

Fig. 1. Spectral distribution for the first four harmonics.

The spectral distributions for different harmonic numbers are presented in Fig. 1; the resulting spectrum is in Fig. 2 (right scale). Both distributions were calculated per an undulator period.

The spectrum (5) depends solely on the parameter K, and has a universal character that allows it to transform into real photon spectra for any values of γ_0, λ_u, and K.

We simulated spectra and polarization of UR for the following parameters [1]:

$$E_0 = \gamma_0 mc^2 = 250\text{GeV}; \ \lambda_u = 1.15\text{cm}; \ N_u = 20000;$$
$$L_u = N_u\lambda_u = 231\text{m}; \ K = 0.92.$$

A collimator with an aperture radius $R_c = 0.7$ mm is placed $L_c = 400$ m from the central point of the undulator [8].

Using the relation (3), we calculated a photon spectrum per unit energy interval, per an undulator period:

$$\frac{dN^{(n)}}{d\hbar\omega^{(n)}} = \frac{\pi\alpha K^2}{2\gamma_0^2\hbar\omega_0} \times \left\{ J_n'^2(nZ) + \left[\frac{\sqrt{2\gamma_0^2\omega_0 n/\omega^{(n)} - 1 - K^2}}{K} - \frac{\gamma_0^2\omega_0 n}{K\omega^{(n)}\sqrt{2\gamma_0^2\omega_0 n/\omega^{(n)} - 1 - K^2}} \right]^2 J_n(nZ) \right\}, \quad (8)$$

where $\omega_0 = 2\pi c/\lambda_u$ (see Fig. 2 (left scale)).

The circular polarization of UR depends on the angle of the outgoing photon and the Stokes parameter ξ_2 characterizes such a polarization component. It can be written as the spectral dependence, using the following relation (4):

$$\xi_2\left(S^{(n)}\right) = \sum_{n=1}^{n_{max}} 2\pi\alpha K N_u \frac{\left[2\left(1+K^2\right)S^{(n)} - n\right]}{\sqrt{S^{(n)}\left[n - S^{(n)}\left(1+K^2\right)\right]}} J_n(nZ) J_n'(nZ) \Big/ \sum_{n=1}^{n_{max}} \frac{dN^{(n)}}{dS^{(n)}}. \quad (9)$$

The dependence of the Stokes parameter ξ_2 on the photon energy for the aforementioned parameters is shown in Fig. 3.

3. The quantum model of the undulator radiation

The process within the classical electrodynamics framework—whereby each treated electron emits ~ 100 photons with energy ~ 10 MeV—needs improvement; this can be had by using the quantum approach.

Other studies [5,6] show the analogy between classical UR and the linear Thomson scattering of the undulator field (approximated by the plane wave) by a relativistic electron. Below, we present further development of such a model for the nonlinear Compton scattering process.

Let us consider the nonlinear Compton backscattering (CBS) process:

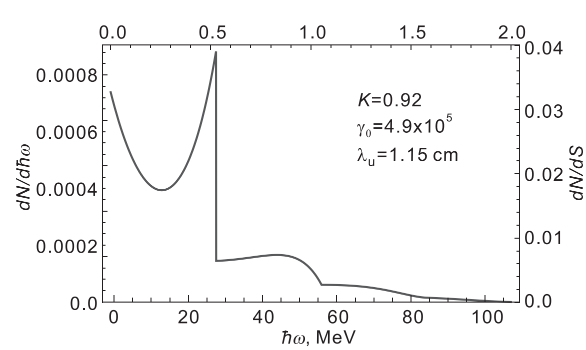

Fig. 2. Resulting spectrum of the summarized harmonics.

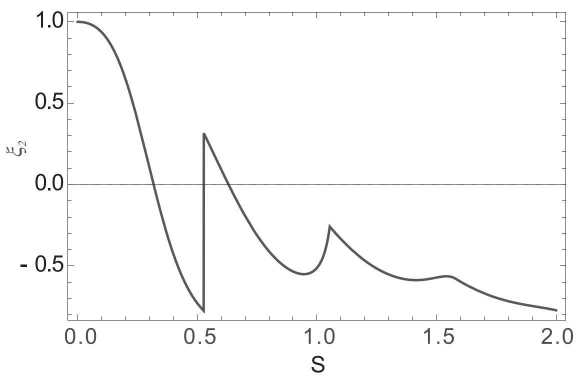

Fig. 3. Circular polarization of UR, summarized over first four harmonics.

$p_0 + nk_0 = p_1 + k_1,$

where the number of 'absorbed' photons n in each interaction is a random quantity with its own distribution law, which is given by the field strength a_0 (the nonlinearity parameter of the process) and initial photon and electron energies (kinematics of the process).

The CBS cross-section is most simply expressed through invariant variables [9]:

$$x_0 = \frac{2p_0k_0}{(mc^2)^2} = \frac{2(1+\beta_0)\gamma_0\hbar\omega_0}{mc^2}, \quad y = 1 - \frac{p_0k_1}{p_0k_0} \approx \frac{\hbar\omega}{\gamma_0 mc^2}. \tag{10}$$

Here, the components of the four-momentum of particles for the head-on collision appear as follows:

$$k_0 = \{\hbar\omega_0, 0, 0, \hbar\omega_0/c\},$$
$$p_0 = \{\gamma_0 mc^2, 0, 0, \gamma_0\beta_0 mc\},$$
$$k_1 = \{\hbar\omega, \hbar\omega\sin\theta/c, 0, \hbar\omega\cos\theta/c\}.$$

In (10), p_0k_0 and p_0k_1 are the products of four vectors, $\beta_0 = \upsilon_0/c; \upsilon_0$ is the velocity of the initial electron.

The cross-section of the nonlinear CBS process with the absorption of n photons is written with invariant variables [9]:

$$\frac{d\sigma^{(n)}}{dy} = \frac{4\pi r_0^2}{x_0 a_0^2} \times \left\{-4J_n^2 + \frac{a_0^2}{2}\left(1 - y + \frac{1}{1-y}\right)\left(J_{n-1}^2 + J_{n+1}^2 - 2J_n^2\right)\right\}. \tag{11}$$

In the latter formula, the quantity J_m^2 is a Bessel function of order $m = n-1,\ n,\ n+1$ of the same argument:

$$z_n = \sqrt{2}na_0\sqrt{\frac{y}{(1-y)nx_0}}\left[1 - \frac{y}{1-y}\frac{(1+a_0^2/2)}{nx_0}\right]. \tag{12}$$

In quantum electrodynamics, the number of absorbed photons n can correlate with the harmonic number of scattered radiation in classical electrodynamics.

It is easy to obtain the expression for a photon spectrum from (11), using the relation $\hbar\omega^{(n)} = \gamma_0 mc^2 y^{(n)}$ (see [5,6]):

$$\frac{dN^{(n)}}{d\hbar\omega^{(n)}} = \frac{\alpha a_0^2 N_0}{4r_0^2}\frac{d\sigma^{(n)}}{d\hbar\omega^{(n)}}, \quad \frac{dN}{d\hbar\omega} = \sum_{n=1}^{n_{max}}\frac{dN^{(n)}}{d\hbar\omega^{(n)}}. \tag{13}$$

where r_0 is the classical electron radius, and N_0 is the number of periods in a wave packet.

To obtain formulas for the static helical undulator, one needs to use substitutions $\omega_0 \to \omega_0/2$ in (10), and $a_0^2 \to 2K^2$ in (11)–(13) [5].

The results of using the formula (13) to calculate the photon spectrum is presented in Fig. 4, after summation over n up to $n_{max} = 4$.

The positions of the spectral peaks are determined by the expression:

$$\hbar\omega_{max}^{(n)} = \gamma_0 mc^2\frac{nx}{1+nx+a_0^2/2} \approx 4\gamma_0^2\frac{n\hbar\omega_0}{1+a_0^2/2}, \tag{14}$$

where $\hbar\omega_0 = \frac{2\pi\hbar c}{\lambda_u}$.

In comparing Figs. 2 and 4, one can conclude that there is excellent coincidence (i.e. with accuracy better than 10^{-3} near the first maximum). The Stokes parameter ξ_2 was calculated by CBS formulas (see, for instance, [10]):

$$\xi_2(\hbar\omega) = \sum_n^{n_{max}} F_2^{(n)}(\hbar\omega) / \sum_n^{n_{max}} F_0^{(n)}(\hbar\omega), \tag{15}$$

where functions $F_0^{(n)}$ and $F_2^{(n)}$ are given by formula (47) of the paper [10]—which also coincides with the results presented in Fig. 3, with the same level of accuracy.

Collimation of the radiation by the aperture θ_c leads to a 'cutoff' of the soft part of the spectrum, and to an increase in the polarization of the final radiation. In the calculations, instead of (4), the following relation must be used [11,12]:

$$\frac{nx}{1+nx+(\gamma_0\theta_c)^2 + a_0^2/2} < \frac{\hbar\omega^{(n)}}{\gamma_0 mc^2} < \frac{nx}{1+nx+a_0^2/2}. \tag{16}$$

Formulas (11)–(15) can be applied for any collision geometry with corresponding calculations of variables x, y, instead of expression (10)--for instance, for the geometry considered by Petrillo [13], where circular polarization ξ_2 was calculated for the linear case.

However, in the case of an undulator with the length L_u—comparable to the distance to the collimator L_c—the collimation angle θ_c is determined by the position of the quanta emission. Thus, to derive a correct calculation of the collimated spectrum and polarization, it is essential to average the classical formulas along the electron trajectory, passing through a 'long' undulator.

The results of this averaging of 'classical' formulas are shown in Fig. 5a; they were obtained for $(\gamma_c\theta_c)_{min} = 0.68$ (emission from the undulator entrance) up to $(\gamma_c\theta_c)_{max} = 1.22$.

In the quantum approach, the following characteristics are simulated: the point of interaction (along the trajectory), and the energy of the emitted photon (and hence the photon emission angle and the energy of the electron after the collision).

The paper [12] shows the step-by-step algorithm used in the CBS simulation process, which takes into account the change in the electron energy during the process of photon emission $N_{tot} \gg 1$.

The simulation results obtained for the same conditions are shown in Fig. 5b. In comparing Figs. 5a and 5b, one sees that the inclusion of the electron energy changes along the path which is lacking in the classical approach, where it leads to an insignificant distortion in the results.

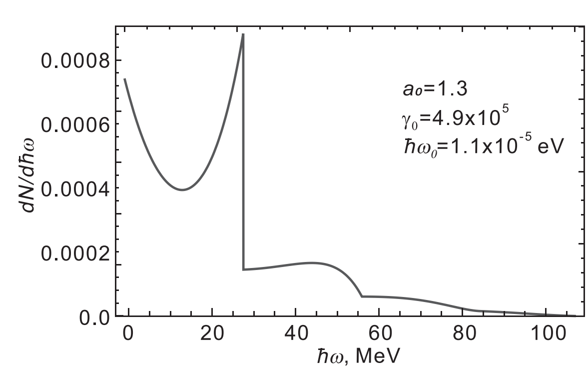

Fig. 4. Photon spectrum calculated in the quantum approach for $n_{max} = 4$.

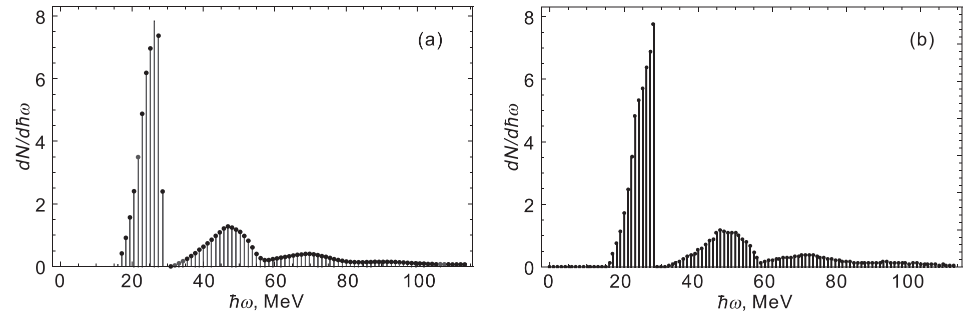

Fig. 5. UR collimated photon spectrum obtained by averaging along the electron trajectory ((a) classical approach, (b) quantum approach).

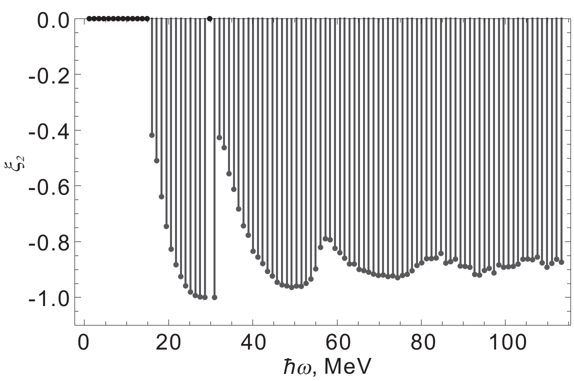

Fig. 6. Polarization of the collimated beam.

The simulation results of the circular polarization degree of the collimated beam are shown in Fig. 6.

One can see that 'rejection' of the soft part of the initial spectrum leads not only to spectrum 'narrowing' but also to an increase in the circular polarization of the final beam (compare Figs. 3 and 6).

4. Conclusion

Intense flux of circularly polarized photons with energy higher than 10 MeV is needed to produce a polarized positron beam at ILC and usually, UR is considered as the most probable source of such quant.

We simulated the spectral and polarization characteristics of collimated UR produced by electrons in a long undulator, by taking both classical and quantum approaches.

In the former case, we calculated UR photon spectra using classical formulas for UR intensity; in the latter, we applied cross-section formulas that describe the nonlinear Compton scattering process. The number of photons accepted by the collimator aperture—which depends on the emission point along the electron trajectory—was simulated in both cases. Comparisons of the results (see Fig. 5a and b) confirm the adequacy of both approaches.

We showed that the classical formulas remain valid when used to calculate UR characteristics for high-energy electrons, when the energy of the emitted photons exceeds ~ 10 MeV.

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