QIPA Quasilinear Equations, Inverse Problems and Their Applications

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The Eurasian Association on Inverse Problems (EAIP)
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Conference program

5 December
Tuesday

9:30 Welcome

Plenary session

Phystech.Bio, 107 / Chair: I. A. Taimanov

10:00 **S. I. Kabanikhin, M.A. Shishlenin** *(ICM&MG SB RAS, Russia)*
Multidimentional analogs of the Gelfand–Levitan-Krein equations

10:35 **M. V. Klibanov** *(University of North Carolina at Charlotte, USA)*
Phaseless inverse scattering and global convergence for inverse problems

11:10 **R.G. Novikov** *(Ecole Polytechnique, France)*
Weighted ray transforms and applications

11:45 **Z.S. Gaeva, A.A. Shananin** *(MIPT, Russia)*
Applying Chebyshev–Markov–Krein algorithm in numerical solving of microphysical hail cloud processes control problem

12:20 Coffee break & lunch

Section "INVERSE PROBLEMS AND TOMOGRAPHY"

Phystech.Bio, 107 / Chair: J. Boman

13:20 **D.V. Lukyanenko, A. Yagola** *(Moscow State University, Russia)*, **Y. Wang**
Regularized inversion of full tensor magnetic gradient data

13:55 **A.D. Agaltsov** *(Max Planck Institute for Solar System Research, Germany)*
Inverse scattering without phase information

14:30 **A.S. Demidov** *(Moscow State University, Russia)*
On the search algorithm for all essentially different solutions of theinverse problem for the Grad-Shafranov equation

15:05 **M. Galchenkova, A.S. Demidov** *(MIPT, Russia)*
Inverse magnetoencephalography problem and its flat approximation

15:40 Coffee break

16:00 **M.N. Demchenko** *(PDMI RAS, Russia)*
Local reconstruction of solution of the wave equation from boundary data

16:35 **A.S. Mikhaylov, V.S. Mikhaylov** *(PDMI RAS, Russia)*
The boundary control method and De Branges spaces

Section "GJB EQUATIONS AND CONVEX ANALYSES"

Main building, 119 / Chair: I.B.Petrov

13:05 **E. S. Polovinkin**
Calculation of Subdifferentials for the Difference of Two Convex Functions

13:55 **M. V. Balashov**
The Lipschitz property of the metric projection in the Hilbert space

14:30 **G.E. Ivanov** *(MIPT, Russia)*
Weak convexity of functions and the infimal convolution

15:05 **M. S. Lopushanski**
Properties of weakly convex sets in asymmetric normed spaces

15:40 Coffee break

Section "VARIATIONAL METHODS"

Main building, 119 / Chair: E. S. Polovinkin

16:00 **S.Z. Adjiev, V.V. Kazantseva, N.N. Fimin, I.V. Melihov, M.A. Negmatov, V. V. Vedenyapin** *(Keldish Inst. Appl. Math., Russia)*
The Hamilton–Jacobi method in the non Hamiltonian situation and Boltzmann extremals

16:35 **V. Zh. Sakbaev** *(MIPT, Russia)*
On the variational description of the averaging quantum dynamical maps

17:15 **A.V. Lapin** *(Kazan Federal University, Russia)*
Domain decomposition and Uzawa-type iterative methods for an elliptic variational inequality

Section "INVERSE PROBLEMS IN ECONOMIC MEASUREMENT"

Laboratory building, 202 / Chair: I.B. Petrov

16:00 **E. G. Molchanov** (MIPT, Russia)
Finding of substitution elasticity in the inverse problem arising in analysis of economic statistics

16:35 **D. A. Alimov** (Moscow State University, Russia)
The study of the Bellman equation in the model of production company functioning in conditions of current assets deficit taking into account influence of debt burden

17:15 **N. I. Klemashev**
(Moscow State University, Russia)
Testing economic data for consistency with the model of two rational representative consumers

Talks

Agaltsov A. D.

Inverse scattering without phase information

(Joint work with A. Gillman, T. Hohage and R. G. Novikov)

We consider the phaseless inverse scattering problem for the Schrodinger equation at fixed energy. The problem consists in determination of an unknown potential under the assumption that only the modulus of the scattering amplitude can be measured directly in an experiment. This limitation is related to the probabilistic interpretation of wavefunction which goes back to Max Born (1926).

We show that a single experiment is not sufficient for the unique determination of the potential, but that it is possible to get rid of this non-uniqueness and recover the potential by making two additional experiments with known background potentials. We introduce an iterative reconstruction algorithm, estimate reconstruction errors and give numerical examples. The talk is based on the results of the work [2] and of the previous works [1,3].

[1] Agaltsov A. D., Novikov R. G., Error estimates for phaseless inverse scattering in the Born approximation at high energies, The Journal of Geometric Analysis, doi:10.1007/s12220-017-9872-6; arXiv:1604.06555v2

[2] Agaltsov A. D., Gillman A., Hohage T., Novikov R. G., An interative approach to monochromatic phaseless inverse scattering, in preparation

[3] Novikov R. G., Explicit formulas and global uniqueness for phaseless inverse scattering in multidimensions, The Journal of Geometric Analysis 26, 346-359, 2016

Alimov D. A.

The study of the Bellman equation in the model of production company functioning in conditions of current assets deficit taking into account influence of debt burden

The production company model with allowance for a working capital deficit and considering influence of a debt load is offered. The study is motivated by new economic operating conditions of production after crisis of 2014. The model is formalized in the form of Bellman's equation for which the sufficient condition of uniqueness of the decision in the chosen class of functions is found. For received conditions economic interpretation is offered. Numerical calculations were performed to analyze the effect of model parameters on the interest rate and the debt discounting option.

Aristova E. N., Karavaeva N. I.

Bicompact schemes for numerical solving of transport problems by HOLO algorithms

The integral-differential transport equation is common description of neutron or uncharged particles transport processes. One of the ways to accelerate iterations due to presence of an integral term in the equation is implementation of HOLO algorithms. At least two (or more) systems of equations are solved: high order (HO) transport equation with respect to distribution function in phase space and time and low order (LO) equations which are direct consequences of the transport equation on some conditions. The first HOLO algorithm was quasi-diffusion method (QDM) suggested by V.Ya.Goldin in 1964. At the present time HOLO algorithms find application for transport problems, plasma etc. In QDM low order equations derived from the transport equation by integration over angle variables defining a direction of particle propagation. These equations are written with respect to macroscopic parameters such as flux and vector flux.

At present work a bicompact schemes of fours approximation order on space and third approximation order on time are used for both HO and LO systems of equations. Bicompact schemes of different approximation orders in space and time (not less than 4 and 3 respectively) for transport equation are investigated in B.V.Rogov works. A bicompact scheme for the quasi-diffusion system of equations with approximation orders matched with these ones for transport equation are presented in this work. The stencil of bicompact scheme includes two points in each space direction and two points in time. The bicompact scheme is constructed in semi-discrete way (method of lines). Time derivatives are remained in differential form, finite differences are used only on space variables. The high order of approximation in space is possible due to widening list of unknown variables and finding not only nodal values of unknown functions but spatial cell averaged values too. SDIRK methods are used for numerical integration of the semi-discrete system over time. Unlike to schemes for transport equation, a boundary problems appear in schemes for the system of quasi-diffusion equations. Economic algorithm for numerical solving of this boundary problem has been suggested.

Balashov M. V.

The Lipschitz property of the metric projection in the Hilbert space

We shall consider when the metric projection on a closed (nonconvex in general) subset from a real Hilbert space is Lipschitz continuous with respect to the point. We shall specify the class of subsets with the Lipschitz property of the metric projection.

Beklemysheva K. A.

Numerical modeling of a low-velocity impact on hybrid composites Abstract

Polymer composites that are being used in aviation are often exposed to low-velocity strikes - hail, gravel, hits during maintenance, etc.). These strikes don't leave visible marks or dents, but lead to the emergence of barely visible impact damage, which severely reduces the residual strength of the whole part. In this work, we propose the application of an existing set of programs based on the grid-characteristic method for the investigation of various parameters that can influence the final strength of hybrid composites with respect to low-speed impacts. The grid-characteristic method proposed in this work was previously used for numerical simulation of polymer composites and helped to obtain a good correspondence to the experiment. Its application for the analysis of failure processes in hybrid composites will allow to modify the structure of the hybrid composite and increase its strength at low-velocity impacts.

Belishev M. I., Vakulenko A. F.

Algebraic approach to tomography problems on manifolds

We develop the algebraic version of the boundary control method, which is an approach to inverse problems based on their relations to control and system theory. In the framework of this version, the reconstruction of a Riemannian manifold via its boundary inverse data is considered. Possible applications of the harmonic quaternion field algebras to 3d elliptic tomography problem are discussed.

Bogdanov L.

Matrix extensions of multidimensional dispersionless integrable systems and SDYM equations on the self-dual background

We present the scheme of matrix integrable extension of multidimensional dispersionless integrable systems, leading to matrix equations on the (geometric) background defined by dispersionless systems. SDYM equations on the self-dual background are considered as an important illustrative example.

Boman J.

Injectivity of the Radon transform with restricted data

For a compactly supported function f in the plane we denote by $Rf(L)$ the value of the Radon transform $Rf(L)$ at the line L, i.e., $Rf(L)=\int_L f ds$, where ds denotes arc length measure on the line L. Given a compact set K in the plane one can ask for which open and connected sets E of lines L it is true that a function f, supported in K, is uniquely determined by $Rf(L)$ for all $L \in E$. This question, as well as a couple of closely related questions, will be discussed in the talk.

Chistyakov V., Kolubin S. A.

Quasilinear integral equation in dynamics of rigid body projectile motion in medium conditions close to real ones

Dual-projective variables were used to trasform radically dynamical equations of projectile motion in medium with density gradient and sophisticated dependence of aerodynamical forces and torques on speed. Account of all the factors above leads to integral equation relatively second derivative of tangent line intercept by inclination. In frames of perturbative approach these equations may be led to quasilinear and solved by iterative method. And inverse Lagrange transformation allows to receive parametrical equation of the projectile trajectory.

Demchenko M. N.

Local reconstruction of solution of the wave equation from boundary data

We consider the problem of determining solution of the wave equation in a domain from Cauchy data given on the boundary. The case of local data is studied: the solution and its normal derivative are given on some part of the boundary on a finite time interval. We provide a reconstruction algorithm based on analytic expressions. Possible reformulations of the problem in consideration are the source identification problem and the problem of recovering initial data in the IBVP for the wave equation. Such problems arise in geophysics (ground-penetrating radar), photoacoustic tomography with limited data, and source identification in the linearized theory of shallow water waves.

Demidov A. S.

On the search algorithm for all essentially different solutions of the inverse problem for the Grad-Shafranov equation

For suppression of plasma instabilities in a tokamak is necessary to know the distribution of electric current $j : \overline{\omega} \ni (x,y) \mapsto j(x,y) \geq 0$ flowing through the cross section ω plasma discharge. In the simplest case, $j(x,y) = f_u(x,y)$, where $f_u(x,y) = f(u(x,y))$. What as regards the functions f and u, it is known only that

$$\Big| \int_\omega f(u(x,y))\, dx dy - 1 \Big| \leq \mu \ll 1, \qquad (1)$$

$$\Delta u = f(u(x,y)) \geq 0, \quad u\big|_{\gamma=\partial\omega} = 0, \quad \sup_{P\in\gamma} \Big| \frac{\partial u}{\partial \nu}(P) - \Phi(P) \Big| \leq \lambda \sup_{P\in\gamma} \big|\Phi(P)\big|. \qquad (2)$$

Here $1/\lambda \gg 1$, and Φ is a given function. It has been shown [1] that for some Φ there are essentially different solutions f_u^1 and f_u^2 of the problem (1)-(2) in the sense that

$$\Big| \frac{\|f_u^1\| - \|f_u^2\|}{\max\{\|f_u^1\|, \|f_u^2\|\}} \Big| \geq \alpha \sim 0.1 \div 0.2, \quad \text{where} \quad \|f_u^j\| \stackrel{def}{=} \max_{(x,y)\in\omega} |f_u^j(x,y)|,$$

and

$$(\hat{x}, \hat{y}) \in \operatorname{absmax} f_u^1 \quad \Rightarrow \quad (\hat{x}, \hat{y}) \in \operatorname{absmin} f_u^2 .$$

The report will consider the finding of all essentially different solutions to the problem (1)-(2).

[1] Demidov A. S. and Savelyev V. V. (2010), Essentially different current distributions in the inverse problem for the Grad-Shafranov equation, Russian Journal of Mathematical Physics, Vol. 17, No 1, pp. 56-65.

Dymarskii Y.

Fibration of the space of potentials and periodic solutions of Korteweg-de Vries equation

We consider the space of stationary Schrodinger Operators on a circle and describe the fibration of this space to hypersurfaces of constant spectral n-th lacuna. Solutions of KdV-equations lie on the fibers, which is homotopic to circle. Therefore, the degree of projection of solution to circle is determined. We prove that the degree is equal to the number n of the eigenvalue.

Dryuma V. S.

On solutions of the flow equations of incompressible liquids

An examples of exact solutions of the Euler system of equations are obtained. Their properties are discussed.

Favorskaya A. V.

About the development of strategies for minimizing of risks of a different nature by full-wave numerical simulation

Full-wave numerical simulation by grid-characteristic method allows to analyze dynamic wave patterns and dynamics of destructions of various structures. This analysis enables geophysicists to study the causes of the appearance of certain effects on seismograms, which is an alternative to solving inverse problems. Also, analysis of dynamic wave patterns can be used to improve algorithms for solving inverse problems. Analysis of the dynamics of destruction of various buildings enables engineers to optimize their structures to minimize the risks of their destruction as a result of negative impacts of natural and man-made character.

Filatova V. M., Pestov L., Nosikova V.

Combined method for detecting small inclusions in the ultrasound tomography problem

The paper is devoted to the problem of determining small sound speed fluctuations in medical ultrasound tomography problem. We studied two-dimensional sound speed model based on model of breast, considered in the papers of N. Duric [1]. The sound speed in the fatty tissue and the glandular tissue are assumed to be known but the boundary "FG" between them is unknown as well as inclusions. The problem we are numerically solved is to find "FG" boundary and inclusions visualization. Moreover we recover sound speed in inclusions.

The method is based on 1) – determination of the internal boundary, 2) – visualization of inclusions using method of Reverse Time Migration and 3) – determination of sound speed in inclusions. The results of numerical simulation are presented.

This work was supported by the Russian Science Foundation under grant 16-11-10027.

[1] Sandhu G. Y., Li C., Roy O., Schmidt S., Duric N., Frequency domain ultrasound waveform tomography: breast imaging using a ring transducer // Physics in Medicine & Biology. 2015. 60, P. 5381-5398

Fomochkina A., Bukchin B.

Application of parallel calculations to speed up the determination of earthquake parameters by a systematic exploration of their values

We consider the source of an earthquake in approximation of instant point shift dislocation. Such a source is given by its depth, the focal mechanism determined by three angles (strike, dip and slip) and the seismic moment characterizing the earthquake intensity. We determine the source depth and focal mechanism by a systematic exploration of 4D parametric space, and seismic moment - by solving the problem of minimization of the misfit between observed and calculated surface wave spectra for every combination of all other parameters.

But this approach in the case of sufficiently high detail of the exploration requires considerable computer time. To speed up the determination of earthquake parameters we apply the methods of parallel calculations. The results of use of such approach to study the deep Okhotsk sea earthquake (2013) are presented.

Gaeva Z. S., Shananin A. A.

Applying Chebyshev-Markov-Krein algorithm in numerical solving of microphysical hail cloud processes control problem

We propose an effective algorithm for derivatives calculation of Chebyshev–Markov–Krein exact estimations. These derivatives are used to solve the problem of controlling microphysical processes in hail clouds.

Galchenkova M., Demidov A. S.

The inverse Magnetoencephalography problem and its flat approximation

Contrary to the prevailing opinion about the incorrectness of the inverse MEEG-problem, we prove that its uniqueness solution in the framework of the electrodynamic system of Maxwell equations~[1]. The solution of this problem is the distribution of $\mathbf{y} \mapsto \mathbf{q}(\mathbf{y})$ current dipoles of brain neurons that occupies the region $Y \subset \mathbb{R}^3$. It is uniquely determined by the non-invasive measurements of the electric and magnetic fields induced by the current dipoles of neurons on the patient's head. The solution can be represented in the form $\mathbf{q} = \mathbf{q}^* + \rho\delta\big|_{\partial Y}$, where $\mathbf{q}^*$ is the usual function defined in Y, and $\rho\delta\big|_{\partial Y}$ is a δ-function on the boundary of the domain Y with a certain density ρ. However, in cases where the conductivity is assumed to be everywhere the same (in the brain, skull, ambient air) and, in addition, it is not possible (or impossible) to record {\it in time} the electric and magnetic inductions, it is impossible to completely find $\mathbf{q}$. Nevertheless, it is still possible to obtain partial information about the distribution of $\mathbf{q} : Y \ni \mathbf{y} \mapsto \mathbf{q}(\mathbf{y})$. This question is considered in detail in a flat model situation.

[1] Demidov A. S. (2017) Unique solvability of the inverse MEEG-problem (to appear)

[2] Stroganova T. A. et al. (2011) EEG alpha activity in the human brain during perception of an illusory kanizsa square, Neuroscience and Behavioral Physiology, V. 41 (2), 130-139.

[3] Hamalainen M. H. et al. (1993) Magnetoencephalography -- theory, instrumentation, and applications to noninvasive studies of the working human brain, Reviews of Modern Physics, V. 65, No 2,413-497.

[4] Demidov A. S. (1973) Elliptic pseudodifferential boundary value problems with a small parameter in the coefficient of the leading operator, Math. USSR-Sb., 20:3, 439-463.

[5] Demidov A. S. (1975) Asymptotics of the solution of the boundary value problem for elliptic pseudo-differential equations with a small parameter with the highest operator, Trudy Moskov. Math. obshchestva, 32, Moscow University Press, M. 119-146 (In Russian).

[6] Demidov A. S., Galchenkova M. A., Kochurov A. S. (2015) On inverse problem magneto-encephalography, Quasilinear equations, inverse problems and their applications. Moscow, 30.11.2015--02.12.2015, Conference handbook and proceedings, p. 22.

Golubev V., Favorskaya A.

Seismic imaging in elastic media: novel approaches

Seismic survey process is the main method for finding oil and gas deposits nowadays. To identify boundaries between different geological layers the specific procedure called migration is used. In common case, it is based on the acoustic approximation for seismic waves propagation that leads to a high level of errors and misestimating.

In this work we discuss novel methods of migration that is based on elastic system of equations. Different approaches like Born approximation, Kirchhoff (Rayleigh) approximation and the grid-characteristic numerical method are used. The results are compared and pros and cons of them are discussed. The research was supported by the grant of the President of the Russian Federation No. MK- 1831.2017.9.

Goncharov F. O.

Injectivity and non-injectivity for weighted Radon transforms

We consider weighted Radon transforms R^d_W (and, in particular, weighted ray transforms along straight lines) with strictly positive weights W. In this talk we recall the results on injectivity and non-injectivity for R^d_W obtained by Quinto (1983), Markoe, Quinto (1985), Finch (1986), Boman (1993) and we present the recent progress in this direction. In particular, we present several recent examples of non-injectivity for R^d_W which significantly extend the aforementioned old results. The talk is based, in particular, on the works [1], [2], [3].

[1] Goncharov F. O., Novikov R. G., An example of non-uniqueness for the weighted Radon transforms along hyperplanes in multidimensions, hal-01583755v2, 2017.

[2] Goncharov F. O., Novikov R. G., An example of non-uniqueness for Radon transforms with continuous positive rotation invariant weights, hal-01593781v1, 2017.

[3] Goncharov F. O., Novikov R. G., An example of non-uniqueness for weighted ray transforms in multidimensions (in preparation), 2017.

Grinevich P. G., Santini P. M.

Direct scattering transform for the periodic self-focusing nonlinear Schrödinger equation

We consider the periodic Cauchy problem for the self-focusing NLS equation assuming that we have a small perturbation of the constant solution. The development of instabilities near the constant solutions is one of the basic models used in the study of rouge waves. In this special situation one has an essential simplification of the theta-functional formulas, and we show, that up to a small corrections, all answers could be reduces to elementary functions (but the elementary functional formulas are different in different regions of the (x,t)-plane).

Hohage, T.

Phase Retrieval and Phase Contrast Tomography: Uniqueness, Stability, and Reconstruction Methods

As opposed to conventional computed tomography, phase contrast tomography does not only yield information on the imaginary part, but also the real part of the refractive index of the sample, which yields good contrast for nanoscale-imaging of living tissues. At each direction of the incident beam a phase retrieval problem has to be solved: The missing phase information of the field in the detector plane have to be reconstructed from the measurable intensities using a-priori information.

Surprisingly, for compactly supported objects the complex-valued field in the exit field is uniquely determined by the amplitude of the field in the detector plane. Even more, we show that the corresponding linearized inverse problem is well-posed. However, the condition number grows exponentially with the Fresnel number. For real-valued refractive indices one has a much more favorable linear growth with the Fresnel number.

Finally, we discuss the solution of phase retrieval problems by regularized Newton method. In particular we show that joint reconstruction of phase and amplitude is possible for moderate Fresnel numbers. Moreover, we present three-dimensional reconstructions from experimental tomographic data using a Newton-Kaczmarz method.

Holodov, A. S.

On the approximation of strongly oscillating data of large volume

Ivanov A., Khokhlov N.

Applying perfectly matched layer boundary condition to wave propagation modeling in layered elastic media

Application of FDTD method on staggered grid to elastic and acoustic waves propagation modeling is considered. Due to axial symmetry of task, equations are rewritten in cylindrical coordinates, because it will reduce dimensionality and computation will be performed faster. PML boundary condition is also should be rewritten in those coordinates. Correct formulation of CPML (convolution PML) in case of axial symmetry is considered. We propose method of sequential correction of computed derivatives according to boundary condition. This procedure independent on type of differential equation and can be applied to any operator.

Ivanov G. E.

Weak convexity of functions and the infimal convolution

In terms of sup-inf convolutions we characterize the class CWC(f) of convolutionally weakly convex functions, generated by the function f. Motivated by Moreau-Yosida and Lasry-Lions regularizations, we consider a subclass of CWC(f), namely, the class of regularly convolutionally weakly convex functions and show that it is exactly the same as previously known class WC(f).

Pankratov L. S.

Nonlinear flow through doudle porosity media in variadle exponent sobolev spaces

We studied the asymptotic behavior of the solution of a nonlinear parabolic equation with nonstandard growth in a ε-periodic fractured medium, where ε is a parameter that characterizes the scale of the microstructure tending to zero. We consider a double porosity type model describing the flow of a compressible fluid in a heterogeneous anisotropic porous medium obeying the nonlinear Darcy law. We assume that the permeability ratio of matrix blocks to fractures is of order $\varepsilon^{p_\varepsilon(x)}$, where $p_\varepsilon(x)$ is a continuous positive function. We obtained the convergence of the solution and a macroscopic model of the problem was constructed using the notion of two-scale convergence combined with the variational homogenization method in the framework of Sobolev spaces with variable exponents.

Petrosyan N. S.

Asymptotic time-behavior of the solution of the mixed problem for quasilinear scalar conservation law

We study the behavior as the time increases indefinitely of the generalized entropy solution of the mixed problem in the quarter of plane for the quasilinear scalar first-order conservation law with a strictly convex flux function. We suppose that the initial function is measurable, bounded and has a limiting mean value that is uniform with respect to translations, the boundary function is constant.

Petrov D. I.

Application of numerical methods in seismic problems in the Arctic

The aim of this work is numerical simulation of wave propagation in the Arctic. The main goal of the research is studying the influence of the Arctic geological medium on longitudinal and transverse velocity components by carrying out numerical experiments.
Keywords: grid-characteristic method, numerical modeling, Arctic seismic exploration

Podoroga A. V., Tikhonov I. V.

The general principal of the stabilization for periodic solutions of the quasi-linear equation of traffic flow

We discuss some mathematical effects that appear by motion of a closed traffic flows on a ring road. The quasi-linear equation of traffic flow with a concave fundamental diagram is studied. We consider weak and periodic (with respect to) solutions that satisfies the standard Oleinik entropy condition. Under the additional assumption of a piecewise linear structure of the fundamental diagram, it is shown that any such solution stabilizes to a moving ordinary wave. We formulate the strict mathematical statement. The results of computer experiments are presented.

Polovinkin E. S.

Calculation of Subdifferentials for the Difference of Two Convex Functions

It is shown that for some classes of functions all epiderivatives and subdifferentials of F. H. Clarke type and P. Michel – J.-P. Penot type and others coincide. Several rules of calculation of subdifferentials for the difference of two convex functions are obtained. Some examples are considered.

Karov D. D., Puro A. E.

Optical Polarimetric Tomography Residual Stresses with Beam Deflection (Grin Articles)

GRIN structures, like rod lenses, laser rods, ball lenses, fiber preforms, optical waveguides, diffractive gratings, etc. are widely used in many fields of optics and optoelectronics. Most of them have an axially symmetric refractive index distribution. Only some works devoted to reconstruction of residual stresses by using polarimetric tensor tomography with beam deflection has been published during this time. All these investigations are based on the measurement of the change in the polarization of light passed through a birefringent medium. Birefringence in GRIN lenses is very weak thus the linearized quasi - isotropic approximation can used as the bases of tensor tomography. In this case, we can separate optical problem on two parts: determination of the ray paths and determination of the change in the polarization of light passed through a birefringent medium. A surprisingly large diversity of optical profiling techniques has been developed during last decades, and first problem has numerous technical solution [1]. Solution of the second problem can be written as two linearized integrals. One of them is connected with the transversal interaction of the two-dimensional vector field and the other with the transversal interaction of the two-dimensional tensor field. From a mathematical point of view we have specal case of tesor tomography with beam deflection [2]. The reconstruction algorithm is based on cicular harmonic decomposition (Cormac – type inversion)[3]. This method of reconstruction is applicable to axially symmetric refractive index and axisymmetric attenuation. In uor case complete reconstruction of the tensor stress field by using sample transilluminatuion in a system of parallel planes is possible only based on additional information. In particula case , the consept of fictitious temperature is used when determinig the residual stress in GRIN lenses. In contrast to the straight light rays polarimetric tomography the optical problem and the inverse thermoelastic problem must be solved together [4].

[1] Yablon A. D. Recent Progress in Optical Fiber Refractive Index Profiling Published in: OSA/OFC/NFOEC 2011. Los Angeles. USA. DOI: 101364/ OFC.2011 OMF1

[2] Monard F. Efficient tensor tomography in fan-beam coordinates, Inverse Probl. Imaging, 10 (2016), pp. 433–459.

[3] Puro A., Garin A. Cormack-type inversion of attenuated Radon transform. Inverse problems. 2013. V.29. N.6 14p. doi:10.1088/0266-5611/29/6/065004

[4] Puro A., Karov D. Inverse problem of thermoelasticity of fiber gratings. Journal of thermal stresses. 2016. V.39. № 5. P. 500 – 512.

Sakbaev V. Zh.

On the variational description of the averaging quantum dynamical maps

The dynamics of quantum system with degenerated Hamiltonian will be studied. To this end the approximating sequence of regularized Hamiltonians and corresponding sequence of dynamical semigroups acting in the space of quantum states will be considered. The regularization semigroups will be investigated as the random variable with values in the space of strongly continuous operator-valued function. The mean values of random semigroups and its properties are investigated. The trajectories of the family of averaging dynamical maps are described by the solutions of some variational problems.

Scherzer O.

Mathematical Imaging with Optical Coherence Tomography and Photoacoustics

In this talk we are discussing mathematical models describing the light propagation in Optical Coherence Tomography (OCT) and Photoacoustic Tomography (PAT) and methods for tomographical imaging. These two imaging modalities allow for visualization of biological specimens of a few millimeters.

In this talk we give an overview on recent microscopic and clinical applications, discuss established mathematical models and inversion (imaging) techniques, as well as recent mathematical trends in these fields.

This talk is based on joint work with W. Drexler (Medical University Vienna), P. Elbau, L. Mindrinos, J. Schmid, C. Shi (University of Vienna).

Shananin A. A.

Inverse problems in economic measurement

The report discusses mathematical problems of economic measurement in the conditions of globalization of world markets.

Sharafutdinov V. A.

Orthogonality relations for a stationary flow of ideal fluid

The work relates to the long standing open question: Do the stationary Euler equations have a non-trivial smooth compactly supported solution in R^3 If such a solution exists, it satisfies an infinite series of integral equations that are called orthogonality relations. For example, if $v = (v_1, v_2, v_3)$ is the velocity vector field, then the functions v_1, v_2, v_3 are pairwise L^2-orthogonal to each other and have coincident L^2-norms.

Shifrin E. G.

Solving the inverse problem of aerpdynamic designing by transformation to a well-posed problem in the hodograph variables

The aerodynamic designing providing the non-separated flow of an ideal perfect or imperfect gas is presented. The Chaplygin transformation of the plane, or axially symmetric potential transonic flow makes it well-posed. The model of ideal gas is adequate by virtue of the existence continuous boundary layer condition. The examples discussed: supersonic Laval nozzles, nozzles with the central body, supersonic nozzle lattices of turbines, subcritical wing profiles, inlet valves of piston engines. Designing turbojets and rocket nozzles is discussed as well.

Shurup A. S., Rumyantseva O. D.

Numerical modeling of functional algorithms for purposes of medical and ocean acoustic tomography problems

Acoustic tomography is the unique approach for studying natural media that are transparent to acoustic waves when direct measurement of medium properties is difficult or impossible. The main characteristics, which influence on propagation of acoustic waves and, as result, which can be reconstructed by acoustic tomography methods, are spatial distributions of sound speed, medium density, absorption coefficient, and vector field of flows. To use algorithms, which allow reconstruction of all mentioned medium characteristics for a wide class of scatterers in a joint tomography scheme, is perspective for solution of the inverse scattering problems in acoustic applications.

Most of the known methods for the solution of acoustic tomography problems are approximate. The linear approximation is generally applied with iteration procedures and regularizations. The general perturbation theory is also considered. On the other hand, there are quite mathematically rigorous (at least, for a rather wide class of scatterers) functional-analytical methods for solving the inverse problems; these methods were initially developed in quantum mechanics. Nowadays, detail investigations based on numerical modeling are required to understand applicability of these methods for acoustics inverse problems.

Possibilities of the functional algorithms [1–5] for the purposes of 2D and 3D acoustic tomography are discussed in the present report. These algorithms take into account the multiple scattering processes and do not require either linearization of the model with iterations or an additional regularization. The joint reconstruction of sound speed, absorption coefficient, medium density and vector field of flows is considered in 2D problem. In comparison with previously reported results, reconstruction of frequency dependence of the absorption coefficient is regarded. The frequency dependence of the absorption coefficient is an additional informative parameter, which can be used in medical applications for early diagnostics of breast cancer.

Results of 3D tomography reconstruction which takes into account multichannel scattering effects are also presented. These results are perspective for the ocean acoustic tomography which takes into account nonadiabatic propagation of acoustic modes in ocean waveguides.

Results of the numerical simulation which show high resolution and good interference resistance of the considered functional algorithms are presented. Thus, the algorithms can be acceptable for practical purposes.

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Stognii P. V.

Numerical modeling of wave processes in the Arctic with ice formations being present

This work discusses seismic replies in the Arctic shelf in models with ice cover and toroses. Much attention is payed to divisible waves in different media.

Taimanov I.

The Moutard transformation of two-dimensional Dirac operators and conformal geometry of surfaces in the four-space

The Moutard transformation for a two-dimensional Dirac operator with a complex-valued potential is constructed. It is showed that this transformation relates the potentials of Weierstrass representations of surfaces related by a composition of the inversion and a reflection with respect to an axis. It is given an analytical description of an explicit example of such a transformation which results in a creation of double points on the spectral curve of a Dirac operator with a double-periodic potential.

Utkin P. S., Sidorenko D. A.

Multiscale modeling of shock wave – dense particles cloud interaction

The work is devoted to the mathematical modeling of shock wave – dense particles cloud interaction. The problem is considered on different scales. The detailed gas dynamics simulations using Cartesian grid method are used to get information about the collective effects during shock wave – particles interaction. The simulations using heterogeneous media mechanics approach (Baer-Nunziato equations) are used to describe the large scale effects such as particles cloud motion. The results of modeling using both approaches are compared with each other and with the available experimental data.

Vedenyapin V., Adjiev S. Z., Kazantseva V. V., Fimin N. N., Melihov I. V., Negmatov M. A.

The Hamilton-Jacobi Method in the Non Hamiltonian Situation and Boltzmann extremals

The hydrodynamic substitution, which is well known in the theory of the Vlasov equation [1–3], has recently been applied to the Liouville equation and Hamiltonian mechanics [4–8]. In [4–6], Kozlov outlined the simplest derivation of the Hamilton–Jacobi(HJ) equation, and the hydrodynamic substitution simply related this derivation to the Liouville equation[7, 8, 10]. The hydrodynamic substitution also solves the interesting geometric problem of how a surface of any dimension subject to an arbitrary system of nonlinear ordinary differential equations moves in Euler coordinates (in Lagrangian coordinates, the answer is obvious). This has created prerequisites for generalizing the HJ method to the non Hamiltonian situation. The H-theorem is proved for generalized equations of chemical kinetics, and important physical examples of such generalizations are considered: a discrete model of the quantum kinetic equations (the Uehling–Uhlenbeck equations) and a quantum Markov process (a quantum random walk). The time means are shown to coincide with the Boltzmann extremes for these equations and for the Liouville equation [9, 11]. This give possibility to prove existence of analogues of action-angles variables in non hamiltonian situation.

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